MA343 Group Theory I – Assignment 3

November 2, 2012, Lecturer: Claas Röver

QUESTION 1. Suppose that $\alpha: G \to H$ is a surjective homomorphism of groups. Let U be a subgroup of H and prove the following.

(a) The pre-image of U under α, i.e. $\{g \in G \mid g^\alpha \in U\}$, is a subgroup of G containing $\ker(\alpha)$.

(b) If U is normal in H, then the pre-image of U is normal in G.

(c) The image of the centre of G under α is contained in the centre of H, i.e. $Z(G)^\alpha \subset Z(H)$.

QUESTION 2. (a) Find an example of a group G with subgroups K and N such that N is normal in G, K is normal in N but K is not normal in G.

(b) Find groups G and H and a surjective homomorphism $\alpha: G \to H$ such that $Z(G)^\alpha \neq Z(H)$.

QUESTION 3. Given a group G and $g \in G$, the *conjugacy class of g in G* is defined as $g^G = \{g^h \mid h \in G\}$, that is the set of all conjugates of g in G.

(a) Determine all the conjugacy classes of D_6.

(b) Let $G = \mathrm{Sym}(X)$, where $|X| = 6$. Make a table that gives for each conjugacy class in G one of its elements, the oder of that element and the size of the conjugacy class of.

QUESTION 4. Determine all groups of order 10 up to isomorphism.

QUESTION 5. Show that $SO_3(\mathbb{F}_2) = \{M \in SL_3(\mathbb{F}_2) \mid M^{-1} = M^t\}$, where M^t is the transpose of M, is isomorphic to D_3.

Hand in your solutions at the beginning of the lecture on Thursday, November 15, 2012.