MA343 Group Theory I – Problem Sheet 6

October 26, 2012, Lecturer: Claas Röver

QUESTION 1. Investigate the group generated by the following 2×2 matrices with complex entries. Find its order, (normal) subgroups and quotients.

$$a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}.$$

QUESTION 2. Define a group G by $G = \langle r, s \mid r^2 = s^2, r^4 = 1, rs = sr \rangle$. Verify that G has order eight and prove that G is neither isomorphic to D_4 nor to the group from Task 1.

QUESTION 3. Recall that a dihedral group is a group which can be generated by two distinct elements of order two.

Show that this definition allows for only one infinite group, up to isomorphism, and determine its centre. What isomorphism types of quotients does this infinite group have?

QUESTION 4. (a) Show that D_6 is isomorphic to $S_3 \times C_2$.

(b) Using a geometric argument, or otherwise, prove that if n is even, then D_n has a quotient isomorphic to $D_{n/2}$ and also a subgroup isomorphic to $D_{n/2}$.

QUESTION 5. Let $G = SL_2(\mathbb{F}_3)$; this is the group of all 2×2 matrices with integer entries modulo 3 whose determinant is one modulo 3. Find the order of G. Show that the centre of G has order 2. Hence, or otherwise, prove that G is not isomorphic to S_4.