MA343 Group Theory I – Assignment 1

September 28, 2012, Lecturer: Claas Röver

QUESTION 1. Let G be a group and H a subgroup of G. Define the notion of a right coset of H in G. Now let G be the group of symmetries of a square and let H be the subgroup generated by one reflection. List all the right cosets of H in G. Also list the cosets of K, the subgroup of G consisting of all rotations.

QUESTION 2. Let $X = \{v, w, x, y\}$ and let $G = \mathrm{Sym}(X)$.

(a) Find all cyclic subgroups of G of order 4.

(b) Show that G has a subgroup H which isomorphic to the symmetric group on a set with three elements.

QUESTION 3. Let G be a group and let $C = \langle z \rangle$ be an infinite cyclic group generated by z. Show that, for every $g \in G$, the map $\alpha_g \colon z \mapsto g$ can be extended to a homomorphism from C to G. Describe the image and kernel of this homomorphism.

QUESTION 4. Let C be a cyclic group of order 6. Find all isomorphisms from C to itself and verify that they form a group.

QUESTION 5. Determine the isomorphism type of the group of 2×2-matrices generated by

$$a = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ and } b = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}.$$

Hand in your solutions at the beginning of the lecture on Friday, October 5, 2012.