MA343 Group Theory I – Problem Sheet 2

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QUESTION 1. Let $a = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$; 2×2 matrices with complex entries. Consider the subgroup $Q = \langle a, b \rangle$ of the group of invertible 2×2 complex matrices.

(a) What is the order of Q? (b) Is Q an abelian group?
(c) Is it true that every element of Q is of the form $a^i b^j$ for some integers $i, j \geq 0$?
(d) How many elements of order two does Q have?

QUESTION 2. Let P be the set of 3×3 matrices of the form $\begin{pmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ with $x, y \in \mathbb{Z}$.

Is P a group under matrix multiplication? Is P an abelian group? Is P a cyclic group? How many elements does one need to generate P?

QUESTION 3. Let G be the group of all symmetries of a regular pentagon.

(a) What is the order of G? (b) What are the orders of elements of G?
(c) Describe all subgroups of G? (d) Does G have a non-cyclic subgroup other than itself?
(e) What is the smallest number of elements needed to generate G?

QUESTION 4. Repeat Question 3 for the group of all symmetries of a regular hexagon.

QUESTION 5. Describe the group of all symmetries of a tetrahedron.