MA286 SAMPLE SOLUTIONS

PROBLEM SHEET 1

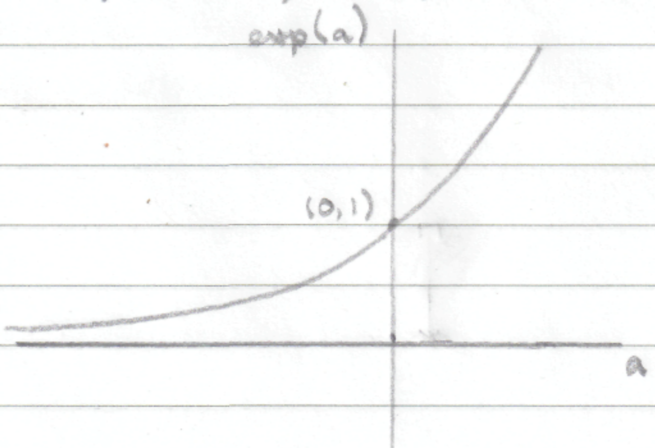
Consider $k(x,y) = e^{-(x^2+y^2)}$

To determine the domain and range of k, recall the exponential function

$\exp(a) = e^a$

which is defined for all $a \in \mathbb{R}$. That is, the domain of $\exp(a)$ is $\mathbb{R}$.

The graph of $\exp(a)$ is sketched below



The range of $\exp(a)$ is the set of possible values of e^a. Thus $\exp(a)$ has range $(0,\infty)$.

Let $a = -(x^2+y^2)$. For every $(x,y) \in \mathbb{R}^2$ we have that $a \in \mathbb{R}$, so $e^a = e^{-(x^2+y^2)}$ is defined for all $(x,y) \in \mathbb{R}^2$. Thus, k has domain $\mathbb{R}^2$.

Notice that $x^2 \geq 0$, $y^2 \geq 0$ and so $-(x^2+y^2) \leq 0$. This implies that if $a = -(x^2+y^2)$, then $a \leq 0$

From the graph of $\exp(a)$ we can see that the range of k is $(0,1]$.

To sketch level curves of k, we set

$$k(x,y) = c$$

where c is in the range of k and solve for x and y. For example, if

$$k(x,y) = e^{-(x^2+y^2)} = 1 ,$$

then

$$\ln\left(e^{-(x^2+y^2)}\right) = -(x^2+y^2) = \ln(1) = 0 .$$

This implies that $x^2 = y^2 = 0$ and so $(x,y) = (0,0)$.

If we choose

$$k(x,y) = e^{-(x^2+y^2)} = 1/2$$

then

$$\ln\left(e^{-(x^2+y^2)}\right) = -(x^2+y^2) = \ln(1/2) = \ln(1) - \ln(2) = -\ln(2)$$

and so $x^2+y^2 = \ln(2) = \left(\sqrt{\ln(2)}\right)^2$. This is the equation of a circle centred at $(0,0)$ with radius $\sqrt{\ln(2)}$.

Thus the level curves for k will be concentric circles centred at $(0,0)$.

Check;

- $f(x,y) = \ln(x^2 - y)$ has domain $\{(x,y) \in \mathbb{R}^2 \mid x^2 > y\}$ and range $\mathbb{R}$.
- $g(x,y) = \dfrac{\cos\left(\frac{x(y-\pi)}{2}\right)}{10}$ has domain $\mathbb{R}^2$ and range $[-1/10, 1/10]$
- $h(x,y) = x^2 - y^2$ has domain $\mathbb{R}^2$ and range $\mathbb{R}$.

3D images;

- top left is $h(x,y)$ - axes indistinguishable
- top right is $f(x,y)$
- bottom left is $g(x,y)$ axes are distinguishable
- bottom right is $h(x,y)$