# How to write Maths

1. Use complete sentences.
2. Don't start a sentence or clause in commas with a symbol.
3. Say what you are doing.

## Sample Solution to Question 1 on MA286 Sheet 2

Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of points in $\mathbb{R}^d$, where $d \geq 2$.

Let $a_n^{(k)}$ be the $k^{th}$ coordinate of $a_n$. So $a_n = (a_n^{(1)}, a_n^{(2)}, \dots, a_n^{(d)})$.

First assume that $\lim\limits_{n\to\infty} a_n^{(k)}$ exists for each $k$ with $1 \leq k \leq d$, and let $\varepsilon > 0$ be given. It follows that there exists $N > 0$ and $a^{(k)}$ for $1 \leq k \leq d$ such that

$$|a_n^{(k)} - a^{(k)}| < \frac{\varepsilon}{\sqrt{d}} \quad \text{for all } n \geq N.$$

With $a = (a^{(1)}, \dots, a^{(d)})$ we then find that

$$\begin{aligned}|a_n - a| &= |(a_n^{(1)}, \dots, a_n^{(d)}) - (a^{(1)}, \dots, a^{(d)})| \\ &= \sqrt{(a_n^{(1)} - a^{(1)})^2 + \dots + (a_n^{(d)} - a^{(d)})^2} \\ &< \sqrt{(\varepsilon/\sqrt{d})^2 + \dots + (\varepsilon/\sqrt{d})^2} = \varepsilon,\end{aligned}$$

which shows that $\lim\limits_{n\to\infty} a_n = a$.

Conversely, assume that $\lim\limits_{n\to\infty} a_n = b = (b^{(1)}, \dots, b^{(d)})$, and let $\delta > 0$. Then there exists $M > 0$ such that

$$|a_n - b| < \delta \quad \text{for all } n \geq M.$$

But $|a_n - b| = \sqrt{(a_n^{(1)} - b^{(1)})^2 + \dots + (a_n^{(d)} - b^{(d)})} \geq |a_n^{(k)} - b^{(k)}|$,

and hence $\lim\limits_{n\to\infty} a_n^{(k)} = b^{(k)}$, as required.