MA 286 SAMPLE SOLUTIONS

PROBLEM SHEET 2

q3. The tangent plane to $f(x,y)$ at the point (x_0, y_0) is given by

$$\left(\partial f/\partial x\,(x_0,y_0)\right)(x-x_0) + \left(\partial f/\partial y\,(x_0,y_0)\right)(y-y_0) = z - z_0$$

where $z_0 = f(x_0, y_0)$.

$f_1(x,y) = x^2 - y^2 + xy$

$\partial f_1/\partial x = 2x + y$, $\partial f_1/\partial y = -2y + x$

The tangent plane to $f_1(x,y)$ at $(1,1)$ is then

$$3(x-1) - (y-1) = z - 1$$

q4. If $\partial f/\partial x\,(x_0,y_0) = 0 = \partial f/\partial y\,(x_0,y_0)$, then (x_0,y_0) is a critical point of $f(x,y)$.

For $f_1(x,y)$ we must have $2x + y = 0$ and $-2y + x = 0$

$2x + y = 0 \Rightarrow x = -y/2$

$-2y + x = 0 \Rightarrow x = 2y$

thus $2y = -y/2$. This is only true at $y = 0$. Now, $x = 2y = 2(0) = 0$ so $(0,0)$ is the only critical point of $f_1(x,y)$.

$\partial^2 f_1/\partial x^2 = 2$ and $\partial^2 f_1/\partial y^2 = -2$

so $(0,0)$ is a local min in x and a local max in y, i.e. $(0,0)$ is a saddle point of $f_1(x,y)$.

Q5. $f(x,y) = \sqrt{20-x^2-7y^2} = (20-x^2-7y^2)^{1/2}$

$\partial f/\partial x = (1/2)(20-x^2-7y^2)^{-1/2}(-2x)$

$$= \frac{-x}{\sqrt{20-x^2-7y^2}}$$

$\partial f/\partial y = (1/2)(20-x^2-7y^2)^{-1/2}(-14y)$

$$= \frac{-7y}{\sqrt{20-x^2-7y^2}}$$

The tangent plane at $(2,1)$ is then

$$z = \sqrt{20-4-7} + \left(-2/\sqrt{20-4-7}\right)(x-2) + \left(-7/\sqrt{20-4-7}\right)(y-1)$$

$$= 3 - 2/3\,(x-2) - 7/3\,(y-1)$$

Near $(2,1)$,

$$f(x,y) \approx 3 - 2/3\,(x-2) - 7/3\,(y-1)$$

so

$$f(1.95, 1.08) \approx 3 - 2/3\,(-0.05) - 7/3\,(0.08)$$
$$= 3 + 1/30 - 14/75$$
$$= 2.84\dot{6}$$