MA286 Analysis I – Problem Sheet 7

October 31, 2012, Lecturer: Claas Röver

QUESTION 1. Calculate the line integral $\int_C f(x,y)\,ds$ in the following cases.

(a) $f(x,y) = x\sin(y)$ and C is the line segment from $(0,3)$ to $(4,6)$.

(b) $f(x,y) = xy^3$ and C is the right half of the circle $x^2 + y^2 = 16$ traversed anti-clockwise.

(c) $f(x,y) = 2y(x+3)$ and C is given by $r(t) = (t^2, 2t)$ with $0 \le t \le 2$.

QUESTION 2. Evaluate the following double integrals.

$$(a) \quad \int_0^1 \int_1^2 (x^2y + 3xy + 2y^2)\,dx\,dy \qquad (b) \quad \int_1^3 \int_0^\pi (x\sin(y) + y\sin(x))\,dx\,dy$$

QUESTION 3. Calculate the volume under paraboloid $z = x^2 + y^2$ and above the region bounded by the curves $y = x^2$ and $x = y^2$.

QUESTION 4. Calculate the volume bounded by the coordinate planes and the plane $3x + 2y + z = 6$.

QUESTION 5. Investigate the function $f(x,y) = \dfrac{x-y}{x^2+y^2+1}$. Find its domain, level curves, range and gradient field. Hint: complete the squares.