

# MA286 Analysis I – Problem Sheet 7

October 31, 2012, Lecturer: Claas Röver

QUESTION 1. Calculate the line integral  $\int_C f(x, y) ds$  in the following cases.

(a)  $f(x, y) = x \sin(y)$  and  $C$  is the line segment from  $(0, 3)$  to  $(4, 6)$ .

(b)  $f(x, y) = xy^3$  and  $C$  is the right half of the circle  $x^2 + y^2 = 16$  traversed anti-clockwise.

(c)  $f(x, y) = 2y(x + 3)$  and  $C$  is given by  $r(t) = (t^2, 2t)$  with  $0 \leq t \leq 2$ .

QUESTION 2. Evaluate the following double integrals.

$$(a) \int_0^1 \int_1^2 (x^2 y + 3xy + 2y^2) dx dy \quad (b) \int_1^3 \int_0^\pi (x \sin(y) + y \sin(x)) dx dy$$

QUESTION 3. Calculate the volume under paraboloid  $z = x^2 + y^2$  and above the region bounded by the curves  $y = x^2$  and  $x = y^2$ .

QUESTION 4. Calculate the volume bounded by the coordinate planes and the plane  $3x + 2y + z = 6$ .

QUESTION 5. Investigate the function  $f(x, y) = \frac{x - y}{x^2 + y^2 + 1}$ . Find its domain, level curves, range and gradient field. Hint: complete the squares.