MA286 Analysis I – Assignment 2

October 15, 2012, Lecturer: Claas Röver

QUESTION 1. Sketch the following parametric curves and calculate their derivatives.

(a) $r(t) = (\sin(t), \cos(3t)),\ 0 \le t \le \pi$

(b) $r(t) = ((1-t^2)\cos(4\pi t), (1-t^2)\sin(4\pi t), t),\ -1 \le t \le 1$

QUESTION 2. Let $P = (1/\sqrt{2},\ 1/2,\ 1/2)$ and $Q = (-1/\sqrt{2},\ 1/2,\ -1/2)$.

(a) Give a parametrisation of the line segment from P to Q.

(b) Verify that P and Q lie on the unit sphere $\mathcal{S}$ and parametrise the short arc of a great circle from P to Q.

(c) Calculate the length of the curves from part (a) and (b).

QUESTION 3. Let $r(t)$ be a parametric curve and let $t = t(s)$ be a reparametrisation. Show that $\frac{dr}{dt}$ and $\frac{dr}{ds}$ are parallel vectors. Explain how their lengths are related. Link this to the velocity, which is a vector, and the speed, which is a real number.

QUESTION 4. Calculate the curvature functions of the following parametric curves.

(a) $r(t) = (t^2,\ \sin(t) - t\cos(t),\ \cos(t) + t\sin(t))$ (b) $r(t) = (t^2,\ 2t,\ \ln(t))$

QUESTION 5. Find an equation for the osculating plane and the osculating circle at $(0, \pi, -2)$ of the curve $r(t) = (2\sin(3t),\ t,\ 2\cos(3t))$

Hand in your written solutions at the **beginning** of the lecture on **Wednesday, 24 October 2012**.