# MA286 Analysis I – Problem Sheet 5

October 10, 2012, Lecturer: Claas Röver

QUESTION 1. (a) If $r(t)$ and $s(t)$ are parametric curves. Then $\frac{d}{dt}(r \cdot s) = r \cdot \frac{ds}{dt} + \frac{dr}{dt} \cdot s$

(b) For $u, v, w \in \mathbb{R}^3$, firstly $(u + v) \times w = u \times w + v \times w$ and if $u$ and $v$ depend on $t$, then $\frac{d}{dt}(u \times v) = \frac{du}{dt} \times v + u \times \frac{dv}{dt}$.

QUESTION 2. Calculate the curvature of

(a) $r(t) = (e^t \cos(t), e^t \sin(t), t)$ at $(1, 0, 0)$.

(b) $r(t) = (t, t^3, \sin(t^2))$ at $(0, 0, 0)$.

(c) the graph of $f(x) = \cos(x)$ at $(0, 1)$.

QUESTION 3. Show that the curvature of the graph of $f \colon \mathbb{R} \longrightarrow \mathbb{R}$ is given by $\frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}}$

QUESTION 4. Find (an equation for) a parabola with curvature 3 at its extreme point. Then find a cubic function, i.e. $f(x) = ax^3 + bx^2 + cx + d$, which has curvature 2 and 3 at its two extrema.

QUESTION 5. Each diagram below shows the graph of a function and the graph of its curvature function. Decide which is which.