MA286 Analysis I – Problem Sheet 4

October 3, 2012, Lecturer: Claas Röver

QUESTION 1. Sketch the following (parametric) curves.

$$p(t) = (t\cos(t), t\sin(t), t) \qquad q(t) = (\cos(t), \sin(2t)) \qquad r(t) = (t, t^2, t^4)$$

QUESTION 2. Calculate the derivatives and unit tangent vectors of the functions in Question 1 and decide which of the curves are traversed at constant speed.

QUESTION 3. Let $P = (2, 1)$ and $Q = (3, 0)$. Find parametrisations of the following curves in the indicated direction.

(a) The line segment joining P to Q.

(b) A semi-circle joining Q to P.

(c) The (i) short and (ii) long arc of a circle of radius 2 joining P to Q.

(d) A spiral winding around a cylinder of radius 1 whose axis is the line $l(t) = (t, t, t)$ and that gains 1 unit height (on the cylinder) per turn. *Hint:* A linear transformation might help.

QUESTION 4. Calculate the length of the following curves.

(a) One turn of a cylindrical helix of radius R which gains H units per turn.

(b) One turn of the cycloide $r(t) = (t - sin(t), 1 - cos(t))$ with $0 \leq t \leq 2\pi$.
Hint: $\cos(2x) = \cos^2(x) - \sin^2(x)$ and $1 = \cos^2(x) + \sin^2(x)$.

QUESTION 5. Reparametrise the following curves by arc length starting at $t = 0$.

$$r(t) = (R\cos(t), R\sin(t)),\ 0 < R = \text{const.} \qquad q(t) = (t + 1, t^2 - 1, 2t^3/3 + 2)$$