MA286 Analysis I – Problem Sheet 2

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QUESTION 1. Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of points in $\mathbb{R}^n$. Show that the sequence has a limit if and only if for each k with $1 \leq k \leq n$ the sequence $a_n^{(k)}$ of the k^{th} coordinate of a_n has a limit.

QUESTION 2. Let $f : \mathbb{R}^2 \to \mathbb{R}$. Prove that $\nabla f(x,y)$ is perpendicular to the level curve through (x,y) provided that $\nabla f(x,y) \neq 0$.

QUESTION 3. For each of the functions f_i below, find the function whose graph is the tangent plane to the graph of f at (a) the point $(1,1)$ and (b) the point $(2,0)$.

$$f_1(x,y) = x^2 - y^2 + xy, \qquad f_2(x,y) = xye^{x-y}, \qquad f_3(x,y) = sin(x)cos(y)$$

QUESTION 4. Determine the critical points of the functions from Question 4 and decide whether they correspond to local extrema or saddle points.

QUESTION 5. Find the linear approximation of the function $f(x,y) = \sqrt{20 - x^2 - 7y^2}$ at the point $(2,1)$ and use it to approximate $f(1.95, 1.08)$.

QUESTION 6. The total resistance of three resistors, connected in parallel with resistamces R_1, R_2 and R_3 is

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

Suppose that $R_1 = 25\Omega$, $R_2 = 40\Omega$ and $R_3 = 50\Omega$, each with a possible error of 0.5%. Estimate the maximum error in the calculated value for R, using differentials.