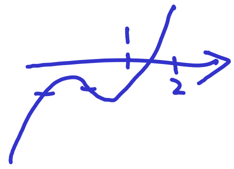
Revision

Newton's Method for approximating zeros:

Example: Find the zero of



$p(x) = x^3 + x^2 - 3$

$p(0) = -3$
$p(-1) = -3$
$p(1) = -1$
$p(2) = 9$

$x_{n+1} = x_n - \dfrac{p(x_n)}{p'(x_n)}$ Choose $x_0 = 1$.

$p'(x) = 3x^2 + 2x$

$$x_{n+1} = x_n - \frac{x_n^3 + x_n^2 - 3}{3x_n^2 + 2x_n}$$

So $x_1 = 1 - \dfrac{1+1-3}{3+2} = 1 + \dfrac{1}{5} = \dfrac{6}{5} = 1.2$

$x_2 = \dfrac{6}{5} - \dots = 1.175$

$x_3 = 1.174559545 59$

$x_4 = x_5 = x_6 = \dots = 1.17455941029$

Integration by Substitution

Find $\displaystyle\int 2x\,(x^2+3)^{50}\,dx$

Put $x^2+3 = u$. So $\dfrac{du}{dx} = 2x$ or $du = 2x\,dx$ and

$$\int 2x\,(x^2+3)^{50}\,dx = \int u^{50}\,du = \frac{1}{51}u^{51} + c = \frac{1}{51}(x^2+3)^{51} + c.$$

Similarly $\int (x+2)\sin(x^2+4x-1)\,dx$

Put $x^2+4x-1=u$. So $du=(2x+4)\,dx$
$=2(x+2)\,dx$

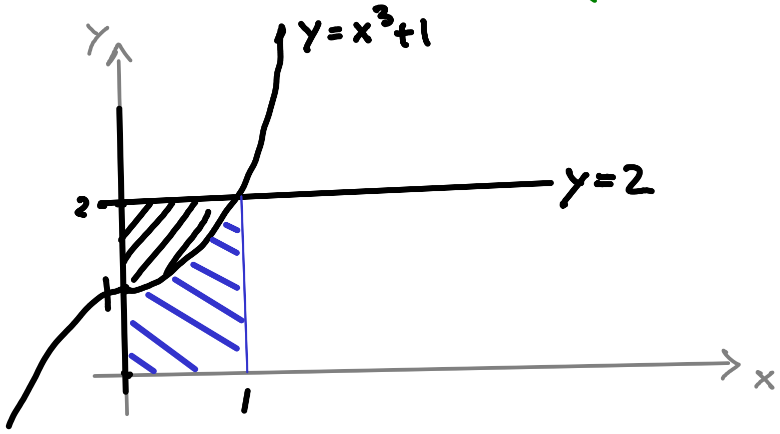
and $\frac{1}{2}du=(x+2)\,dx$. Then

$$\int (x+2)\sin(x^2+4x-1)\,dx = \int \frac{1}{2}\sin(u)\,du$$

$$= -\frac{1}{2}\cos(u)+c = -\frac{1}{2}\cos(x^2+4x-1)+c$$

Volume of revolution

Sketch the area bounded by $y=x^3+1$, $y=2$ and the y-axis and calculate the volume of the solid obtained by rotating it around the x-axis.



The volume is

$$\int_0^1 \pi\left(\underset{\substack{\uparrow\\ \text{upper}\\ \text{bound}}}{2^2}-\underset{\substack{\uparrow\\ \text{lower}\\ \text{bound}}}{(x^3+1)^2}\right)dx = \pi\int_0^1 4\,dx - \pi\int_0^1 (x^3+1)^2\,dx$$

disc of rad. 2 and height 1

blue area rotated.

$$= \pi \left[4x \right]_0^1 - \pi \int_0^1 (x^6 + 2x^3 + 1)\, dx$$

$$= 4\pi - \pi \left[\frac{1}{7}x^7 + \frac{1}{2}x^4 + x \right]_0^1$$

$$= 4\pi - \pi \left(1 + \frac{1}{2} + \frac{1}{7} \right) = 4\pi - \pi \left(\frac{14 + 7 + 2}{14} \right)$$

$$= \pi \left(4 - \frac{23}{14} \right) = \frac{33}{14}\pi$$