More Webwork Problems

Problem 9: The general logistic equation is

$$\frac{dP}{dt} = kP(1-P)$$ where P is the ratio of

$\frac{\text{current population}}{\text{maximal population}}$, so $0 \le P \le 1$

If we want to measure absolute popopation, say N, then then equation becomes

$$\frac{dN}{dt} = kN\left(1 - \frac{N}{K}\right)$$, where K is capacity.

From the problem text we get $k=2$, $K = 7750$

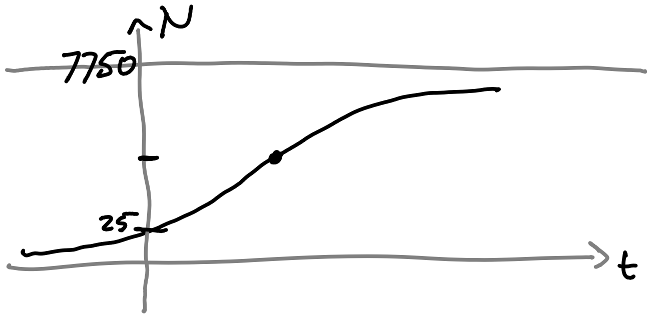
and $N(0) = 25$.

From lecture 20 we have that $N(t) = \frac{K}{1+Ae^{-kt}}$.

In the problem this is $N(t) = \frac{7750}{1+Ae^{-2t}}$.

Now $N(0) = \frac{7750}{1+A} = 25 \implies \frac{7725}{25} = A = 309$

and we get $N(t) = \frac{7750}{1+309e^{-2t}}$



The point of inflection is where $\frac{dN}{dt}$ has a maximum

$\frac{dN}{dt} = 2N\left(1 - \frac{N}{7750}\right)$ which is quadratic equation in N

This a parabola whose max is in the middle between its zeros which are a $N=0$ and $N=7750$. This means that at $N = \frac{7750}{2} = 3875$ the rate of infection starts to decrease.

Problem 3: The sentence "the coffee cools at $3°C$ per minute when the coffee is $70°C$" means

$$\underset{\text{cools}}{-3} = k(70-22) \; . \; \text{So } k = \frac{-3}{48} = -\frac{1}{16}$$

Finding the general solution of

$$\frac{dT}{dt} = k(T-A), \qquad A = \text{const.}$$

Put $S = T - A$. Then $\frac{dS}{dt} = \frac{dT}{dt} = kS$, and we know that

$$\underset{T(t)-A}{\underset{\parallel}{S(t)}} = Ce^{kt}, \quad \text{for some constant } C.$$

and we find $T(t) = A + Ce^{kt}$.