Q4 Assignment 1

A particle travels along a straight line with velocity

$$v(t) = t^2 e^{-2t} \quad \text{at time } t \text{, measured in } \frac{m}{s}$$

How many meters has the particle travelled during the first t seconds?

Sol^n: This is the Net Change Theorem, since $v(t)$ is the rate of change of the position $p(t)$.

So we want $p(t) - p(0) = \int_0^t v(x)\,dx$

$\boxed{\text{Integration by parts: } \int f'g\,dx = fg - \int f g'\,dx}$

$$= \int_0^t \underbrace{x^2}_{f}\underbrace{e^{-2x}}_{g'}\,dx = \Big[\underbrace{x^2}_{f}\cdot\big(\underbrace{-\tfrac{1}{2}e^{-2x}}_{g}\big)\Big]_0^t - \int_0^t \underbrace{2x}_{f'}\big(\underbrace{-\tfrac{1}{2}e^{-2x}}_{g}\big)\,dx$$

$\boxed{\frac{d}{dx} e^{-2x} = -2e^{-2x}}$

$$= -\frac{1}{2}t^2 e^{-2t} - 0 + \int_0^t \underbrace{x}_{u}\,\underbrace{e^{-2x}}_{v'}\,dx$$

$$= -\frac{1}{2}t^2 e^{-2t} + \Big[-\frac{1}{2}x e^{-2x}\Big]_0^t - \int_0^t -\frac{1}{2}e^{-2x}\,dx$$

$$= -\frac{1}{2}t^2 e^{-2t} - \frac{1}{2}t e^{-2t} + \int_0^t \frac{1}{2}e^{-2x}\,dx$$

$$= -\frac{1}{2}e^{-2t}(t^2 + t) + \Big[-\frac{1}{4}e^{-2x}\Big]_0^t$$

Finding this mistake is tricky!

$$= -\frac{1}{2}e^{-2t}(t^2 + t) - \frac{1}{4}e^{-2t} + \frac{1}{4}$$

$$= -\frac{1}{2}e^{-2t}\left(t^2 + t + \frac{1}{2}\right) + \frac{1}{4}$$

Check: $\frac{d}{dx}\left(-\frac{1}{2}e^{-2t}\left(t^2+t+\frac{1}{2}\right)+\frac{1}{4}\right)$

$$= e^{-2t}\left(t^2+t+\frac{1}{2}\right)-\frac{1}{2}e^{-2t}(2t+1)$$

$$= e^{-2t}\left(t^2+t+\frac{1}{2}-t-\frac{1}{2}\right)=t^2e^{-2t} \checkmark$$