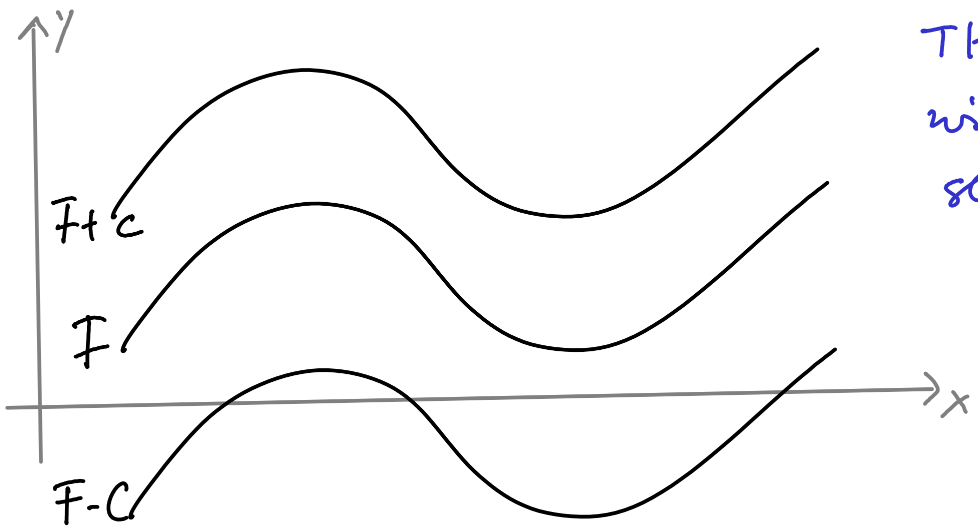
Antiderivatives

Definition: An antiderivative of a function f is a function F such that $F' = f$.

Examples:

$f(x)$	antiderivative of $f(x)$
x	$\frac{1}{2}x^2 + 5$
$\sin(x)$	$-\cos(x)$
e^x	e^x
$\ln(x)$	$x\ln(x) - x$ (not obvious)
$x^{-1} = \frac{1}{x}$	$\ln(x)$
x^m	$\frac{1}{m+1}x^{m+1}$ $\quad m \neq -1$

Theorem: Any two antiderivatives of f differ by a constant. In other words, if $F' = G' = f$, then $G - F = \text{const}$.



Three graphs with the same slope at every x.

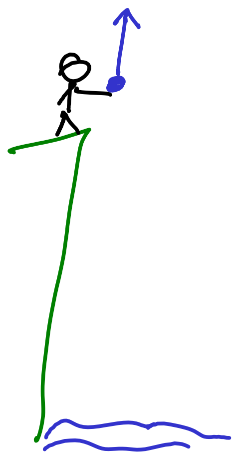
Proof: Suppose $G' = F' = f$. Then

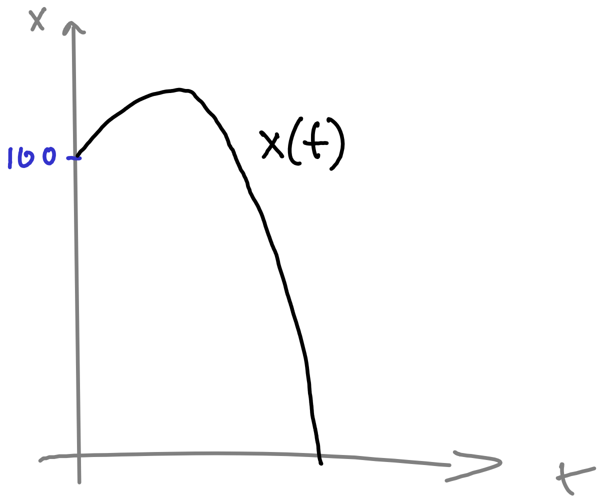
$(G-F)' = G' - F' = 0$, so $G-F$ has a horizontal tangent everywhere and hence is itself a horizontal line. But that means $G - F = C = \text{const.}$

Antiderivatives come up in science, whenever it is easier to measure the derivative of the desired function. Examples are the velocity of a particle, whose antiderivative is the position.

Consequence: Having found an antiderivative F of f, then the most general antiderivative of f is $F + C$, where C is an arbitrary constant.

Example: Standing on the cliffs of Moher 100 m above sea level, you throw a stone vertically upwards with an initial speed of 3 m/s. When does the stone hit the water.





Gravity pulls the stone down : $a(t) = -9.8\, m/s^2$

(a downwards accellerating force)

$v'(t) = a(t)$ gives

$$v(t) = -9.8\, m/s^2\, t + C$$

$$v(0) = C = 3\, m/s$$ (our initial velocity)

$$x'(t) = v(t) = 3\, m/s - 9.8\, m/s^2\, t$$

$$x(t) = 3\, m/s\, t - 4.9\, m/s^2\, t^2 + D$$

$$x(0) = D = 100\, m$$ (initial position)

So $x(t) = 100 + 3t - 4.9t^2$.

Now we need a zero of $x(t)$, i.e.

$$t^2 - \frac{3}{4.9} t - \frac{100}{4.9} = 0$$

$$t = \frac{3}{9.8} \pm \sqrt{\frac{9}{(9.8)^2} + \frac{100}{4.9}} = 4.8$$ (in seconds)