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# MA140 Mid-Term Exam - Semester I 2012/13

Date: October 30, 2012 Time: 9.10am-10am Lecturer: Dr C. Röver

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1. Calculate the following limits, if they exist.

(a)  $\lim_{x \rightarrow 0} \frac{x - \sin(x)}{x(x+2)}$

(b)  $\lim_{n \rightarrow \infty} \frac{(n-2)(2n+3)}{n^2+n-6}$

2. Calculate  $\frac{df}{dx}$  in the following two cases.

(a)  $f(x) = \frac{(x-1)^2(x+2)^3}{(x+1)^2(x-4)}$  Hint: use logs.

(b)  $f(x) = \cos^2(x^3 - 3x^2 + 5x - 3)$

3. Let  $f(x) = 9xe^x$ .

(a) Determine the  $x$  and  $y$  intercepts of the graph of  $f$ .

(b) Determine all local maxima and minima, if any, of  $f$ .

(c) Determine all, if any, points of inflection of  $f$ .

(d) Sketch the graph of  $f(x)$ .

(e) How many asymptotes does the graph of  $f$  have?

4. (a) An oil tank in the shape of a vertical cylinder of radius  $4m$  is filled with oil. The height of the oil in the tank is increasing at a rate of 8 centimetre per minute. At what rate, in cubic meter per minute, is the oil being pumped into the tank?

(b) Recall that the amount  $f(t)$  of a radioactive material at time  $t$  satisfies  $\frac{df}{dt} = kf$ .

Which percentage of a radio-active material is left after 100 years, if its half-life is 1200 years? The half-life is the time it takes for 50% of the material to decay.

5. Find the equation for the tangent line to the graph of  $f(x) = x^3 - x - 4$  at the point where  $x = -1$ . Then determine all points where this tangent line and the graph of  $f$  intersect.