

Revision

Exam 2010/11

Q3 a) (i) $y' = 3x^2 \cos(x^3) = \frac{dy}{dx}$ (Chain Rule)

(ii) $\frac{dy}{dx} = \frac{\sqrt{x}' \cos(x) - \frac{1}{2} x^{-\frac{1}{2}} \sin(x)}{x}$ (Quotient Rule)

(iii) $y = \frac{(x+1)\sqrt{x+2}}{(x+3)\sqrt{x+4}}$

$$\ln y = \ln(x+1) + \frac{1}{2} \ln(x+2) - \ln(x+3) - \frac{1}{2} \ln(x+4)$$

$$\frac{y'}{y} = \frac{1}{x+1} + \frac{1}{2(x+2)} - \frac{1}{x+3} - \frac{1}{2(x+4)}$$

$$\text{So } y' = \frac{(x+1)\sqrt{x+2}'}{(x+3)\sqrt{x+4}} \left(\frac{1}{x+1} + \frac{1}{2(x+2)} - \frac{1}{x+3} - \frac{1}{2(x+4)} \right).$$

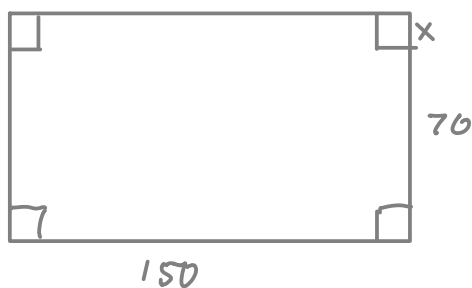
(b) The area of the rectangle is $A = lw$ where l is the length and w is the width, all depending on t .

Then $\frac{dA}{dt} = \frac{dl}{dt} w + l \frac{dw}{dt}$. Since $\frac{dA}{dt} = 0$, we get

$$\frac{dl}{dt} = - \frac{l}{w} \frac{dw}{dt} = - \frac{16}{12} 3 = -4 \text{ and so the width}$$

is decreasing at 4 m/s .

4(a)



$$V = (70 - 2x)(150 - 2x)x$$

$$= \underline{\underline{(70x - 2x^2)(150 - 2x)}}$$

$$\frac{dV}{dx} = (70 - 4x)(150 - 2x) - 2x(70 - 2x) \quad \checkmark$$

$$= 10500 - 740x + 8x^2 - 140x + 4x^2$$

$$= 12x^2 - 880x + 10500 = 0$$

$$3x^2 - 220x + 2625 = 0$$

$$x = \frac{110}{3} \pm \sqrt{\left(\frac{110}{3}\right)^2 - \frac{2625}{3}} = \begin{cases} \frac{175}{3} = 58\frac{1}{3} \\ \frac{45}{3} = 15 \end{cases}$$

$58\frac{1}{3}$ is not the answer; the box won't exist.

So $x = 15$ and then $V = 40 \times 120 \times 15 = 72,000$.

(b) Let N be the number of cells, then $\frac{dN}{dt} = kN$

and we know that then $N(t) = A e^{kt}$ for some constant A . We are given

$$N(0) = A = 500 \quad \text{and} \quad N(24) = 500 e^{24k} = 800.$$

$$e^{24k} = \frac{8}{5} \quad \text{and} \quad 24k = \ln\left(\frac{8}{5}\right)$$

$$\text{and so } k = \frac{1}{24} \ln\left(\frac{8}{5}\right).$$

Hence another 12 hours later, there are

$$N(36) = 500 e^{\frac{3}{2} \ln\left(\frac{8}{5}\right)} = 500 \left(\frac{8}{5}\right)^{\frac{3}{2}} = 1011 \text{ cells.}$$