Some other Taylor series

Problem: Find the Taylor series for $\sin(x)$ at 0.

Sol^n: $f(x) = \sin(x)$, $f'(x) = \cos(x)$

$f''(x) = -\sin(x)$, $f'''(x) = -\cos(x)$

now it repeats. Since $\sin(0) = 0$ and $\cos(0) = 1$, we get

$$\sin(x) = 0 + x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + - \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1}$$

Covered Chapters in Glyn James Book

Background Chapters 1 & 2.

Limits: Chapter 7.5, 7.8, 7.9

Differentiation: Chapter 8.1 – 8.5

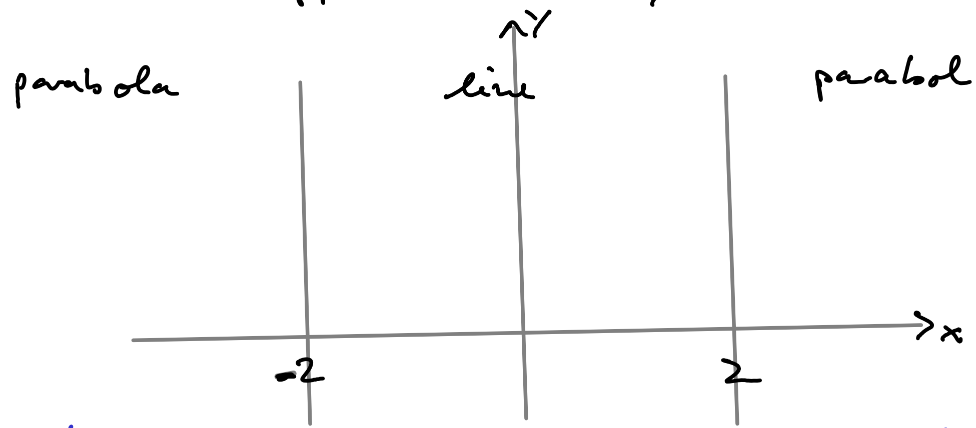
Integration: Chapter 8.7. – 8.10

Revision

Problem,

Let $f(x) = \begin{cases} 2x^2 - n, & x < -2 \\ mx + l, & -2 \le x \le 2 \\ kx^2 + 1, & 2 < x \end{cases}$

Find values for l, m, n and k such that g is continuous and differentiable everywhere.



Since each piece is a polynomial, it is continuous. So to make g continuous, we need to match the pieces

$\lim_{x \to -2^-} 2x^2 - n = \lim_{x \to -2^+} mx + l$ or $8 - n = -2m + l$

$\lim_{x \to 2^-} mx + l = \lim_{x \to 2^+} kx^2 + 1$ or $2m + l = 4k + 1$

In order to make g differentiable, we need the same slope at the meeting points.

$\lim_{x \to -2^-} 4x = m$, i.e. $-8 = m$

$m = \lim_{x \to 2^+} 2kx$, i.e. $m = 4k$

So $m = -8$, $k = -2$, $l = 9$ and $n = -17$.

Problem: Evaluate $\lim_{x \to 0} \frac{x \sin(x)}{1-\cos(x)}$. (Exam (2010/11) Q2(a))

Use L'Hospital's Rule:

$$\lim_{x \to 0} \frac{x \sin(x)}{1-\cos(x)} = \lim_{x \to 0} \frac{\sin(x) + x\cos(x)}{\sin(x)}$$

$$= \lim_{x \to 0} \frac{2\cos(x) - x\sin(x)}{\cos(x)} = \frac{2-0}{1} = 2.$$

Q2 b (Summer 2010/11)

Here $f(x) = x^4 - 8x^3 + 10$. So

$f'(x) = 4x^3 - 24x^2 = 4x^2(x-6)$ and

$f''(x) = 12x^2 - 48x = 12x(x-4)$

Critical points are where $f'(x) = 0$, i.e. $x=0$ and $x=6$.

Since $f''(0) = 0$, we have a point of inflection

From $f''(6) > 0$ we see that there is a local minimum at $x=6$.

f is decreasing when $f' < 0$, i.e. $(-\infty, 6]$

Points of inflection are at 0 and 4

f is concave up if $f'' > 0$, $(-\infty, 0]$ and $[4, \infty)$

Now $f(0)=10$, $f(4)=-246$ and $f(6)=-422$ and the graph looks as follows.