Continuing yesterdays problem

Recall that we tried to decide wether

$$\int_2^\infty \frac{x}{x^3-3x-2}\,dx \quad \text{exists or not.}$$

Using partial fractions we got

$$\int_2^\infty \frac{x}{x^3-3x-2}\,dx = \frac{1}{9}\int_2^\infty \frac{-2x-2+3}{(x+1)^2} + \frac{2}{9}\int_2^\infty \frac{1}{x-2}\,dx$$

$$= \frac{1}{9}\Big[-\ln\left((x+1)^2\right)\Big]_2^\infty + \frac{1}{9}\int_2^\infty \frac{3}{(x+1)^2}\,dx + \Big[\frac{2}{9}\ln(x-2)\Big]_2^\infty$$

$$= \frac{1}{9}\Big[-2\ln(x+1) - \frac{3}{x+1} + 2\ln(x-2)\Big]_2^\infty$$

$$= \frac{1}{9}\Big[2\ln\left(\frac{x-2}{x+1}\right) - \frac{3}{x+1}\Big]_2^\infty$$

$\ln(a)-\ln(b)$
$=\ln\left(\frac{a}{b}\right)$

$$= \frac{1}{9}\left[\lim_{L\to\infty}\Big(2\ln\big(\underbrace{\tfrac{L-2}{L+1}}_{\to 1}\big) - \underbrace{\frac{3}{L+1}}_{\to 0}\Big) - \lim_{\varepsilon\to 2^+}\Big(2\ln\left(\frac{\varepsilon-2}{\varepsilon+1}\right) - \frac{3}{\varepsilon+1}\Big)\right]$$

$$= -\frac{1}{9}\underbrace{\lim_{\varepsilon\to 2^+}\Big(2\ln\left(\frac{\varepsilon-2}{\varepsilon+1}\right)\Big)}_{\text{does not exist.}} + \frac{1}{9}$$

The problem is that roughly speaking we are integrating $\frac{1}{x^2}$ from 0 to ∞, which does not exist.

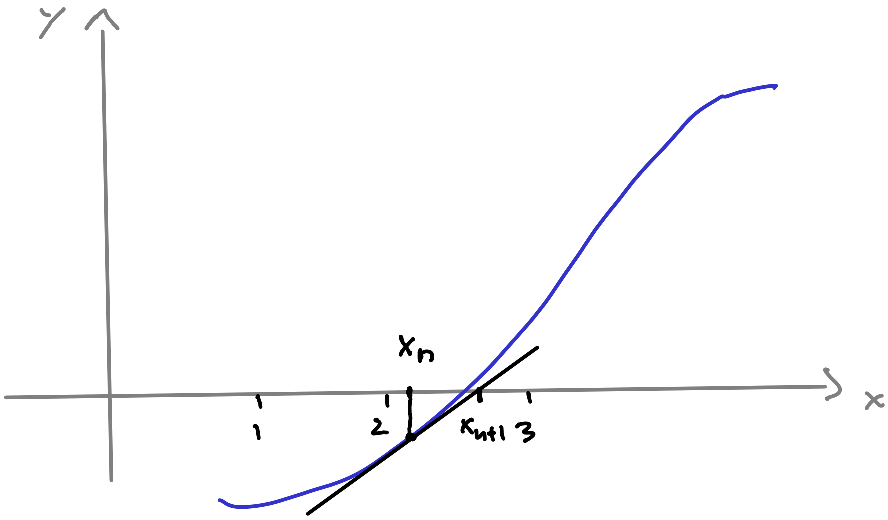
Because 2 is the right most root of x^3-3x-2.

However, the calculation above shows that

$$\int_3^\infty \frac{x}{x^3-3x-2}\,dx = -\frac{1}{9}\ln\left(\frac{1}{4}\right) + \frac{1}{9}\cdot\frac{3}{4} = \frac{2}{9}\ln(2)+\frac{1}{12}$$

Numerical Methods

Newton's Method for finding zeros of $f(x)$.



This is iterative process calculating the next approximation x_{n+1} from the last approximation x_n.

1. Find the first approximation x_1 close to where the real zero is.
2. From x_n calculate x_{n+1} as the zero of the tangent line to the graph of f at the point $(x_n, f(x_n))$

The tangent line at x_n is given by

$$t(x) = f'(x_n)\,x + b \quad \text{s.t. } t(x_n) = f(x_n)$$

$$t(x_n) = f'(x_n)\,x_n + b = f(x_n)$$

So $b = f(x_n) - f'(x_n)\,x_n$ and

$$t(x) = f'(x_n)\,x + f(x_n) - f'(x_n)\,x_n$$

Hence $t(x) = 0$ implies

$$x = \frac{f'(x_n)\,x_n - f(x_n)}{f'(x_n)} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This gives Newton's approximation scheme:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

<u>Example:</u> let $f(x) = x^3 - 2$. Find its root up to 4 decimal places.

<u>Sol^n:</u> $f'(x) = 3x^2$. Also $f(1) = -1$ and $f(\frac{3}{2}) = \frac{11}{8}$,

So the zero of f is between 1 and $3/2$.

Here $x_{n+1} = x_n - \dfrac{x_n^3 - 2}{3x_n^2} = \dfrac{2\,(x_n^3 + 1)}{3\,x_n^2}$

Choose $x_1 = 1$. Then $x_2 = \dfrac{4}{3}$, $x_3 = \dfrac{2\left(\frac{64}{27} + 1\right)}{\frac{16}{3}} = 1.263\overline{8}$

$x_4 = 1.2599$. Not much happens after this.