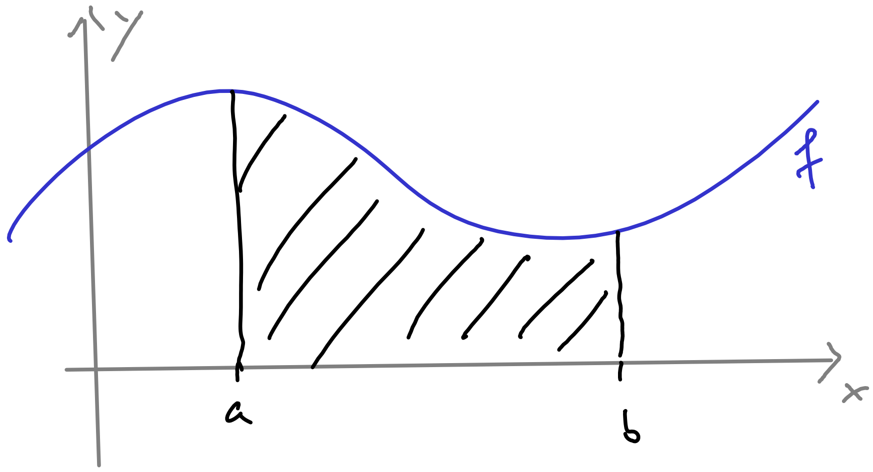
Integration continued

Recall that we write

$$\int_a^b f(x)\,dx$$

for the definite integral from a to b of $f(x)$, which is defined as the limit of approximations of the area (for $f(x)>0$) between $x=a$, $x=b$, the x-axis and the graph of f.



The Fundamental Theorem of Calculus

For a continuous function f, we have

$$\frac{d}{dt}\left(\int_a^t f(x)\,dx\right) = f(t).$$

This means that integrating and then differentiating is the identity operation.

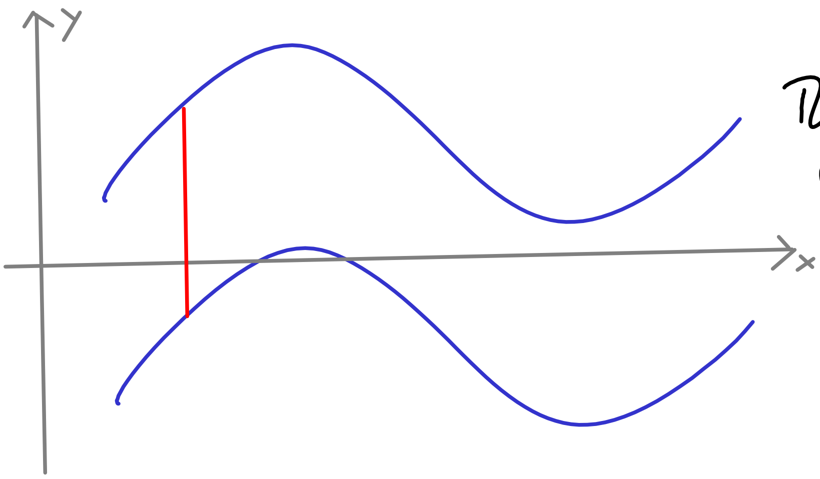
This already allows us to calculate some integrals.

<u>Definition</u>: Given a function $f(x)$, we say that $F(x)$ is an anti-derivative of $f(x)$ if $\frac{dF}{dx}(x) = f(x)$.

The Fundamental Theorem of Calculus says:

$$\int_a^x f(t)\,dt \quad \text{is an anti-derivative of } f.$$

Note: Since derivative is slope, there are many anti-derivatives for a given f.



These two graph have the same slope at every x.

Any two anti-derivatives of f differ by a constant.

We write $\int f(x)\,dx$ for an anti-derivative of f.

Basic integrals

$f(x)$	$\int f(x)$	
x^n	$\frac{1}{n+1} x^{n+1}$	for $n \neq -1$
x^{-1}	$\ln\|x\|$	(by definition)
$\sin(x)$	$-\cos(x)$	
$\cos(x)$	$\sin(x)$	
e^x	e^x	
$\ln(x)$	$x\ln(x) - x$	(came from MAXIMA)

Check: $\frac{d}{dx}(x\ln(x) - x)$

$= \ln(x) + x\frac{1}{x} - 1 = \ln(x)$.

Basic Rules

let f and g be functions, and C a constant

1. $$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$$

2. $$\int C f(x)\,dx = C \int f(x)\,dx$$

3. For $a < b < c$: $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

④ If $F(x) = \int f(x)\,dx$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Examples : Find $\int_0^3 -2x+7\,dx$

$$\int -2x+7\,dx = -x^2 + 7x + \underset{\uparrow \text{ a constant}}{c} = F(x)$$

$$\int_0^3 -2x+7\,dx = \left[-x^2 + 7x + c\right]_0^3$$

$$= F(3) - F(0)$$

$$= (-9 + 21 + c) - (c)$$

$$= 12$$