Recall arcsin is the inverse function of sin.

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \underset{\arcsin}{\overset{\sin}{\rightleftarrows}} [-1, 1]$$

So $\arcsin(\sin(x)) = x$ and $\sin(\arcsin(x))$.

We found $\frac{d}{dx}\left(\arcsin(x)\right) = \frac{1}{\cos(\arcsin(x))}$.

The books say $\frac{d}{dx}\left(\arcsin(x)\right) = \frac{1}{\sqrt{1-x^2}}$.

What's going on here?

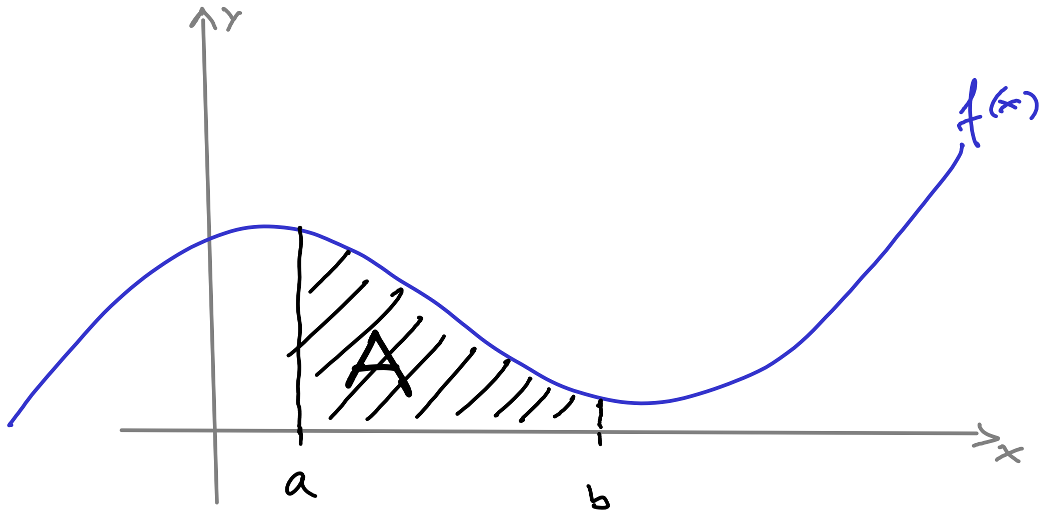
Note that for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$, we have $\cos(t) \geq 0$

So $\cos(t) = \sqrt{\cos^2(t)} = \sqrt{1-\sin^2(t)}$ and with $t = \arcsin(x)$

$$\cos(\arcsin(x)) = \sqrt{1-\sin^2(\arcsin(x))} = \sqrt{1-x^2}.$$

INTEGRATION

Integration can be seen as the inverse operation to differentiation, or as a limiting process for calculating areas, volumes, lengths of curves, work (pysics).



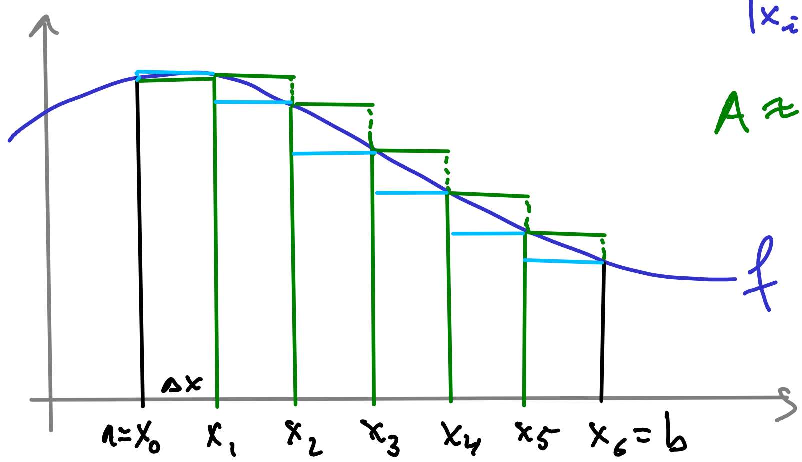
What is the area A below the graph of f ($f(x) \geq 0$), the x-axis and the two lines $x=a$ and $x=b$?

Idea: Split the area A into rectangles (approximately) of the same width. Then take the limit when the number of rectangles goes to infinity.

$|x_i - x_{i-1}| = \Delta x$

$A \approx f(x_0)\Delta x + f(x_1)\Delta x + \dots$
$\dots + f(x_5)\Delta x$

$A \approx f(x_1)\Delta x + \dots + f(x_6)\Delta x$



On the interval $[a,b]$, if we choose N rectagles, then $\Delta x = \dfrac{b-a}{N}$ and the area is approximately

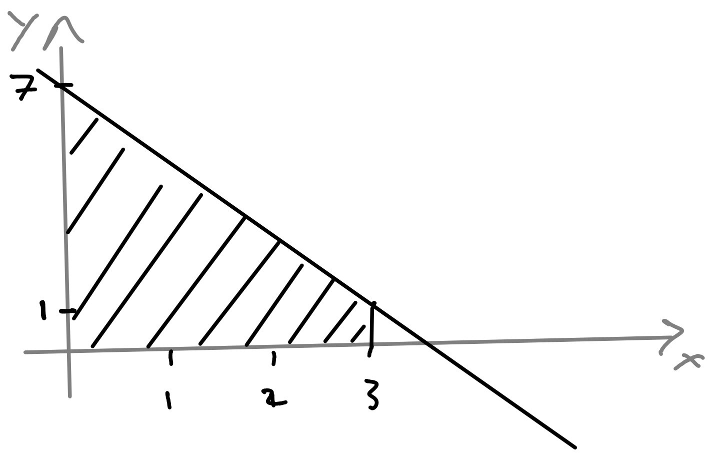
$$A \approx \Delta x \Big(f(a) + f(a+\Delta x) + f(a+2\Delta x) + \dots + f(a+(N-1)\Delta x) \Big)$$

$$= \Delta x \sum_{n=0}^{N-1} f(a+n\Delta x)$$

The limit when $N \to \infty$ is called the integral of f from a to b and written as

$$\int_a^b f(x)\,dx$$

Example: Find the area under the graph of $f(x) = -2x + 7$ between $x=0$ and $x=3$



Here $b=3$, $a=0$. Let's start with $N=3$, then $\Delta x = 1$

$$A_3 \approx \Delta x \big(f(0) + f(1) + f(2) \big) = 7 + 5 + 3 = 15$$

$N = 6$, then $\Delta x = \frac{1}{2}$ and

$$A_6 \approx \frac{1}{2}\left(f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2) + f(\tfrac{5}{2})\right)$$

$$= \frac{1}{2}\left(f(0) + f(1) + f(2)\right) + \frac{1}{2}(6 + 4 + 2)$$

$$= \frac{15}{2} + 6 = 13.5$$

$-2x+7$

$N = 12$, then $\Delta x = \frac{1}{4}$

$$A_{12} = \frac{1}{2}A_6 + \frac{1}{4}\left(f(\tfrac{1}{4}) + f(\tfrac{3}{4}) + f(\tfrac{5}{4}) + f(\tfrac{7}{4}) + f(\tfrac{9}{4}) + f(\tfrac{11}{4})\right)$$

$$= 6.75 + \frac{1}{4}\left(\frac{13}{2} + \frac{11}{2} + \frac{9}{2} + \frac{7}{2} + \frac{5}{2} + \frac{3}{2}\right)$$

$$= 6.75 + \frac{1}{8}(48) = 12.75$$