Example: A cup of coffee in room at 20°C cools from 80°C to 50°C in five minutes. How long will it take to cool down to 40°C?

Newton: A hot object cools at a rate proportional to the difference between its temperature and the room temperature.

Let $T(t)$ be the temperature of the object at time t.

Newton: $\frac{dT}{dt} = k\left(T(t) - T_0\right)$, where T_0 is the room temperature.

We also know $T(0) = 80$ and $T(5) = 50$.

Question is: For which t is $T(t) = 40$?

$\frac{dT}{dt} = k(T - T_0)$. Let's put $S = T - T_0$.

Then $\frac{dS}{dt} = \frac{dT}{dt}$ and

$\frac{dS}{dt} = kS$ whose solution is $S = Ae^{kt}$.

So $T = S + T_0 = \boxed{Ae^{kt} + T_0 = T(t)}$ $\quad T_0 = 20$

Now use $T(0) = A + T_0 = 80$ or $A = 80 - T_0 = 60$,

and then $T(5) = 60e^{5k} + 20 = 50$

or $e^{5k} = \frac{1}{2}$ or $5k = \ln\left(\frac{1}{2}\right)$.

So $k = \frac{-\ln(2)}{5}$. To sum up: $T(t) = 60e^{-\frac{\ln(2)}{5}t} + 20$.

Finally: Solve $40 = 60e^{-\frac{\ln(2)}{5}t} + 20$

or $\frac{1}{3} = e^{-\frac{\ln(2)}{5}t}$

or $-\ln(3) = -\frac{\ln(2)}{5}t$

and so $t = \frac{5\ln(3)}{\ln(2)} \approx 7.925$.

Another example: A radioactive material has a half life of 1200 years. Given a sample of the material, what percentage remains after 10 years?

Fact: The quantity of radioactive material decays at a rate proportional to the amount of radioactive material.

Half life, is the time it takes for half the material to decay.

Let $y(t)$ be the amount of radioactive material at time t.

We know that $y(1200) = \frac{1}{2}y(0)$. Also $\frac{dy}{dt} = ky$,

so $y(t) = Ae^{kt}$, $y(0) = A$, $y(1200) = \frac{1}{2}A = Ae^{1200k}$,

which gives us $-\ln(2) = 1200k$, or $k = \frac{-\ln(2)}{1200}$.

Next: $y(10) = Ae^{-\frac{\ln(2)}{1200}10} = Ae^{-\frac{\ln(2)}{120}} = A\left(\frac{1}{2}\right)^{\frac{1}{120}}$

$= A * 0.9942$

This says that after 10 years 99.42% of the material remain.