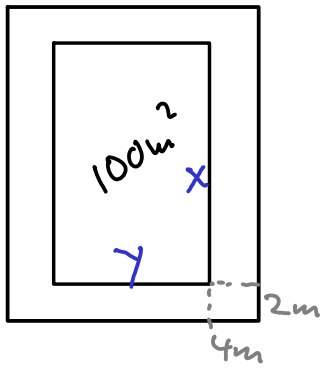
Problem: A billboard is to have $100\ m^2$ of printed area with 2m margins at top and bottom and 4m margins on the sides.



Find the dimensions if the total area is to be minimal.

$xy = 100$

total area $A = (x+4)(y+8)$

$= xy + 4y + 8x + 32$

$= 132 + 4y + 8x$

Also $y = \frac{100}{x}$. So $A(x) = 132 + \frac{400}{x} + 8x$

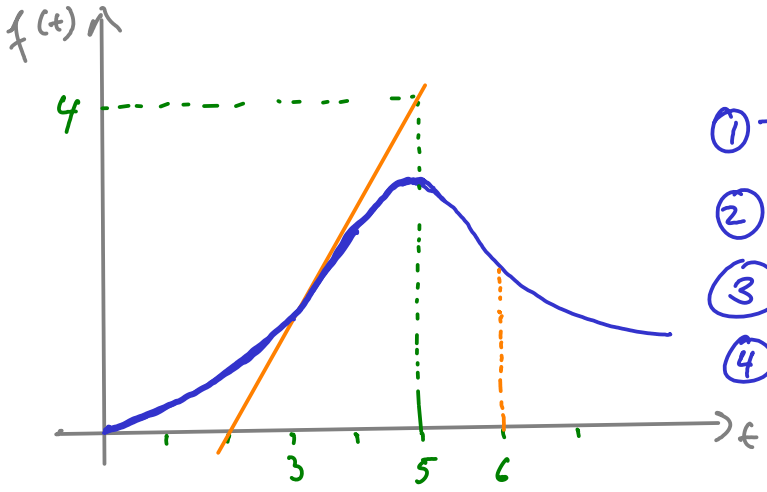
$\frac{dA}{dx} = 8 - \frac{400}{x^2}$ and $\frac{dA}{dx} = 0 = 8 - \frac{400}{x^2}$

means $8x^2 = 400$ or $x^2 = 50$ or $x = \pm\sqrt{50}$

So the dimensions are $\sqrt{50}$ by $2\sqrt{50}$.

Understanding / plotting curves

Example: A car is driving on a long straight road. At time t, its distance from the start is $f(t)$ and the graph of f is



Determine when

① The speed is positive

② The speed is negative

③ Car is accelerating

④ The fastest speed the car reaches.

$f'(t)$ + ++ ... + + 0 − − − − −

Speed is derivative of distance.

Acceleration is rate of change of speed, i.e second derivative of f

Asw: ① positive speed $0 \le t \le 5$

② negative speed $5 \le t$

③ accelerating $0 \le t \le 3$

④ fastest speed = slope of tangent at point of inflection, which is $4/3$

Problem: Sketch the graph of $y = x e^{-x^2/2} = f(x)$

x-axis intercept: $y = 0 \iff x = 0$

y-axis intercept: $x = 0$, so $(0, 0)$

$$f'(x) = e^{-x^2/2} + x e^{-x^2/2}(-x) = (1 - x^2) e^{-x^2/2}$$

critical points: $f'(x) = 0 \iff 1 - x^2 = 0$, or $x = \pm 1$

$$\lim_{x \to \infty} f(x) = 0, \quad \lim_{x \to -\infty} f(x) = 0$$