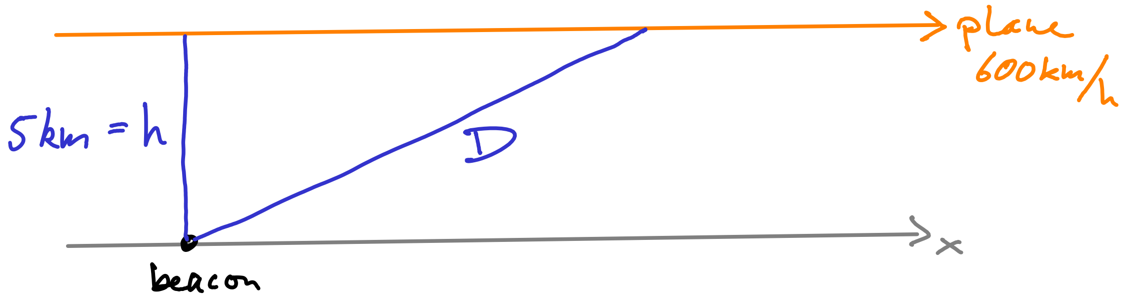
More applications

The derivative can be thought of as "rate of change".

Problem: An aircraft is flying horizontally at a speed of 600 km/h. How fast is the distance between a radio beacon and the aircraft increasing 1 minute after the plane passes 5 km above the beacon?



Sol^n: We need to find $\frac{dD}{dt}$ at $t=1$.

When $t=1$, then $x=10$ and

$$D^2 = 5^2 + x^2$$

Note: $D = D(x)$ but $x = x(t)$.

By the chain rule:

$$\frac{dD}{dt} = \frac{dD}{dx}\frac{dx}{dt}$$

So $D = \sqrt{5^2 + x^2} = (25 + x^2)^{1/2}$

and
$$\frac{dD}{dt} = \frac{1}{2}(25+x^2)^{-1/2}\, 2x\, \frac{dx}{dt}$$

horizontal speed

$$= 10\,(25+x^2)^{-1/2}\, x$$

Calculate in minutes!

$$= \frac{10x}{(25+x^2)^{1/2}}$$

So when $t=1$, $x=10$ and

$$\frac{dD}{dt}(1\,\text{min}) = \frac{10\times 10}{(125)^{1/2}} = \frac{20}{\sqrt{5}} \qquad \left[\frac{\text{km/min}\ \text{km}}{\text{km}} = \text{km/min}\right]$$

Problem: How fast does the volume of a balloon increase, when its radius increases at a constant rate of 2cm per minute?

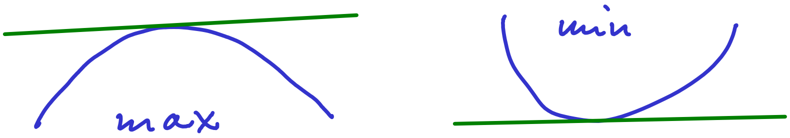
Sol^n: The volume V is given by

$$V = V(r) = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt} = 8\pi r^2$$

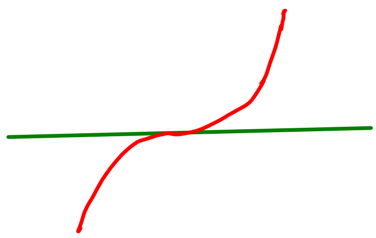
This depends on the current radius!

Derivatives can be used to find where the function increases, decreases, has a maximum or minimum, has a point of inflection.

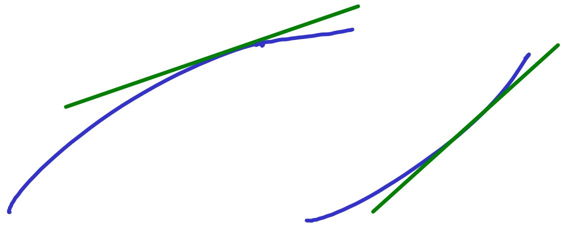


horizontal tangent, i.e. slope $= 0$

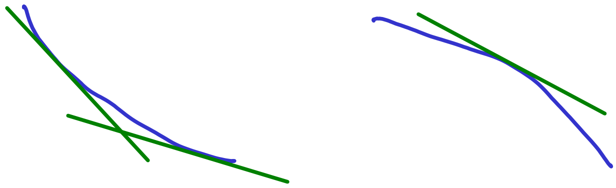
Warning:



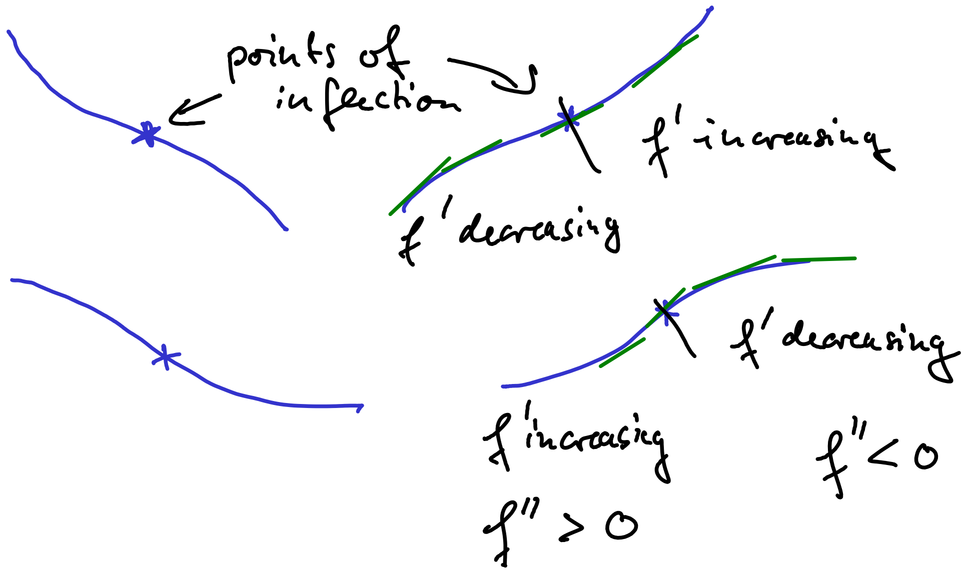
also has horizontal tangent.



f increasing is equivalent to positive slope; $f' > 0$



f decreasing is equivalent to negative slope; $f' < 0$



In particular, at a max of f, f' is decreasing and $f'' > 0$. At a min of f, f' is increasing and $f'' < 0$.