A bit more on asymptotes and discontinuities

An **asymptote** for a graph of a function is a vertical or horizontal line which the graph approaches, gets arbitrarily close but never touches.

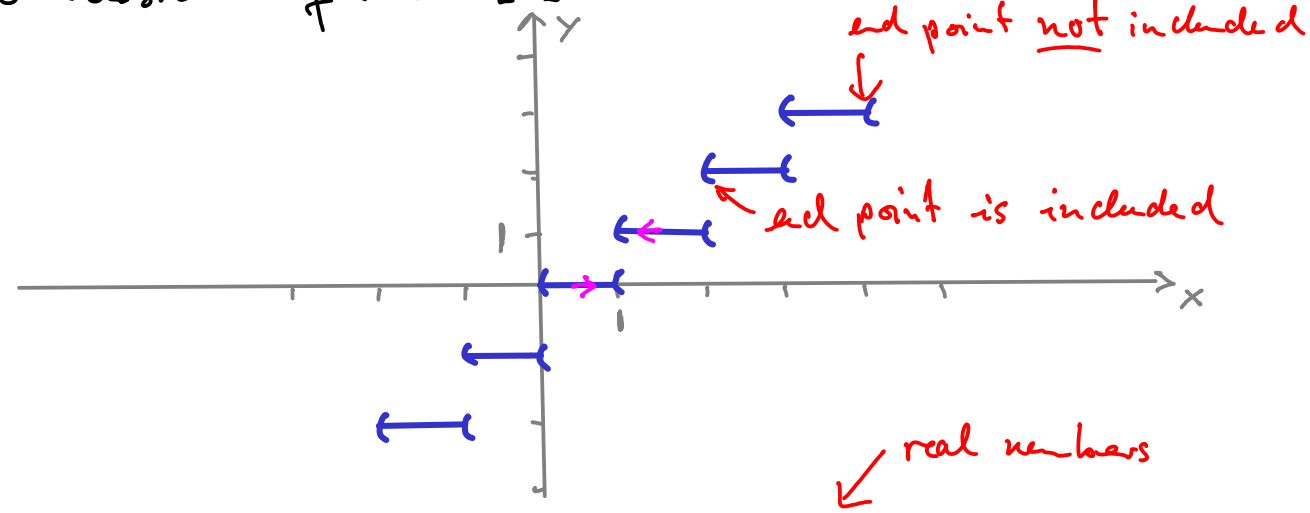
A **discontinuity of a function f** is a "gap", i.e. a value x_0 s.t. $f(x_0)$ is not defined or

$$\lim_{x \to x_0} f(x) \text{ does not exist.}$$

Examples: For a real number x, let $\lfloor x \rfloor$ be defined as the largest integer less or equal to x.

$\lfloor 2.781 \rfloor = 2$; $\lfloor -5.6 \rfloor = -6$; $\lfloor 123.56891 \rfloor = 123$

Now consider $f(x) = \lfloor x \rfloor$



This function is defined for all $x \in \mathbb{R}$ and has discontinuities at $x \in \mathbb{Z}$ ← integers

NOTATION: $\lim_{x \to 1^-} \lfloor x \rfloor = 0$ and $\lim_{x \to 1^+} \lfloor x \rfloor = 1$

← from left ← from right

Example: Consider $f(x) = \dfrac{2x^2}{(x-1)(x+2)}$.

Sketch the graph of f.

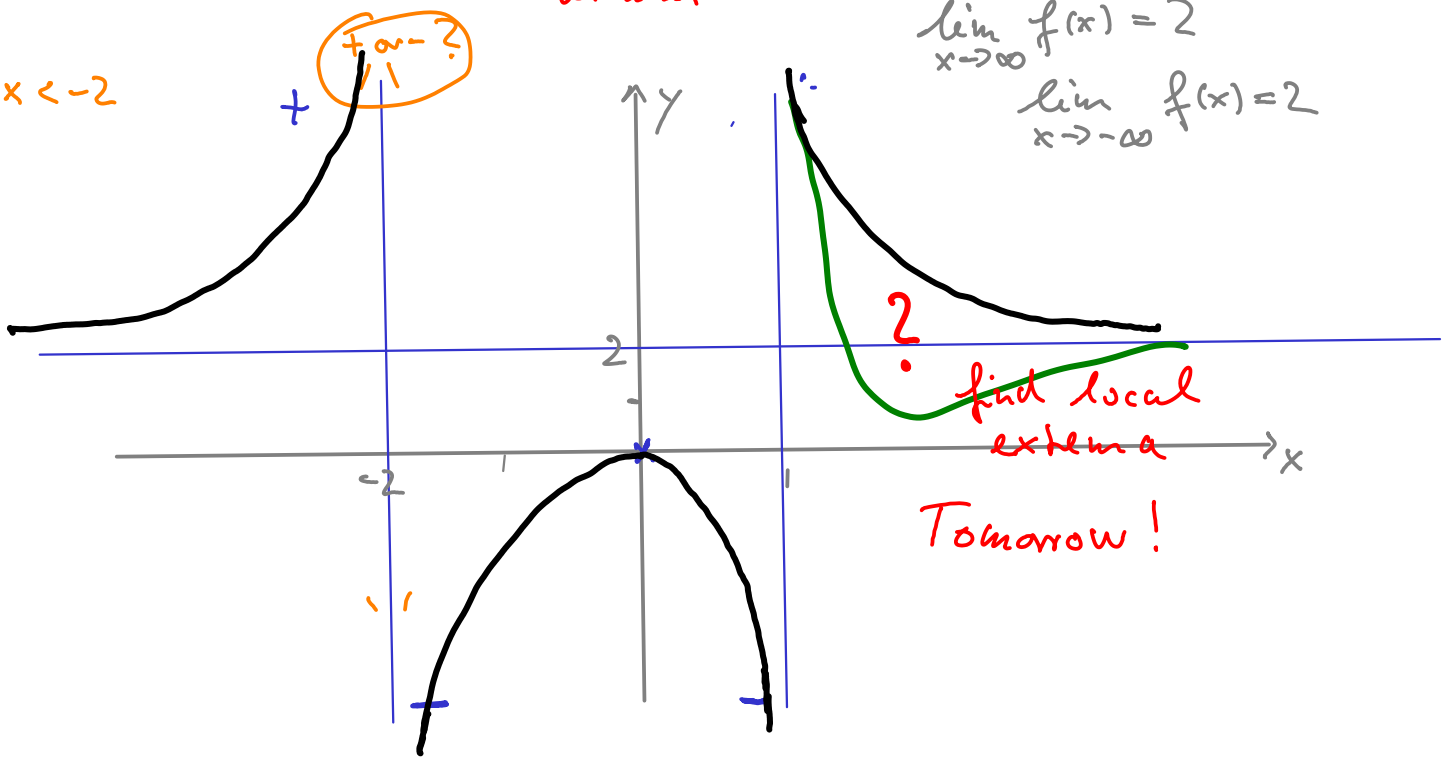
Domain $(f) = \mathbb{R} \setminus \{1, -2\}$ ← without

x-intercept: solve $f(x) = 0$

So $x = 0$ is x-intercept

$\lim_{x \to \infty} f(x) = 2$

$\lim_{x \to -\infty} f(x) = 2$



Tomorrow!