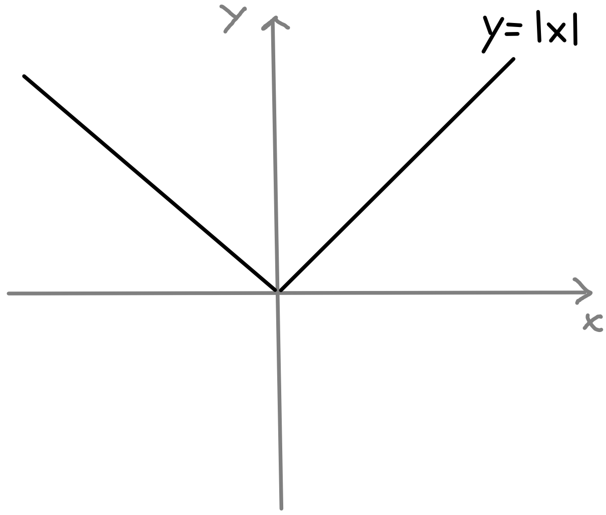
More on limits

The absolute value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Evaluate $\displaystyle\lim_{x\to 0} \frac{x}{|x-1| - |x+1|}$

For $x \neq 0$ but x close to 0, we have

$$\frac{x}{|x-1| - |x+1|} = \frac{x}{(1-x)-(x+1)} = \frac{x}{1-x-x-1} = \frac{x}{-2x} = -\frac{1}{2}$$

So $\displaystyle\lim_{x\to 0} \frac{x}{|x-1|-|x+1|} = -\frac{1}{2}$

Limit definition

Limits are defined for sequences of number and they may or may not exist.

A sequence is an infinite ordered collection of numbers. Ex. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

write this as $a_n = \frac{1}{n}$, $n \in \mathbb{N}$

We say that the sequence $(a_n)_{n\in\mathbb{N}}$ has limit L if for all sufficiently large n we have a_n close to L.

In maths speak: For every $\varepsilon > 0$ there is an $N \in \mathbb{N}$ s.t. for all $n \geq N$ $\underbrace{|a_n - L|}_{\text{this is distance between } a_n \text{ and } L} < \varepsilon$

In this case we write $\lim\limits_{n\to\infty} a_n = L$.

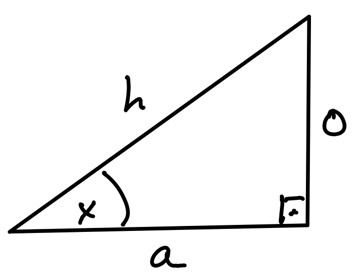
Ex: • $\lim\limits_{n\to\infty} \frac{1}{n} = 0$

• $\lim\limits_{n\to\infty} \frac{2n^2+4n}{n^3+6} = 0$ Trick: $\frac{2n^2+4n}{n^3+6} \cdot \frac{1/n^3}{1/n^3}$

$$= \frac{2/n + 4/n^2}{1 + 6/n^3} \xrightarrow{n\to\infty} \frac{0+0}{1+0} = 0$$

• $\lim\limits_{n\to\infty} \frac{3n^2+6n-2}{4n^2-7n+3} = \lim\limits_{n\to\infty} \frac{3+6/n-2/n^2}{4-7/n+3/n^2} = \frac{3}{4}$

Limits of trigonometric functions



For $0 \leq x \leq \frac{\pi}{2}$ $(90°)$

$\sin(x) = \frac{o}{h}$ $\tan(x) = \frac{o}{a}$

$\cos(x) = \frac{a}{h}$ $= \frac{\sin(x)}{\cos(x)}$

Useful identities: $\cos^2(x) + \sin^2(x) = 1$

$$\frac{a^2}{h^2} + \frac{o^2}{h^2} = \frac{a^2+o^2}{h^2} \underset{\text{Pythagoras}}{=} \frac{h^2}{h^2} = 1$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

Circle interpretation of sine and cosine