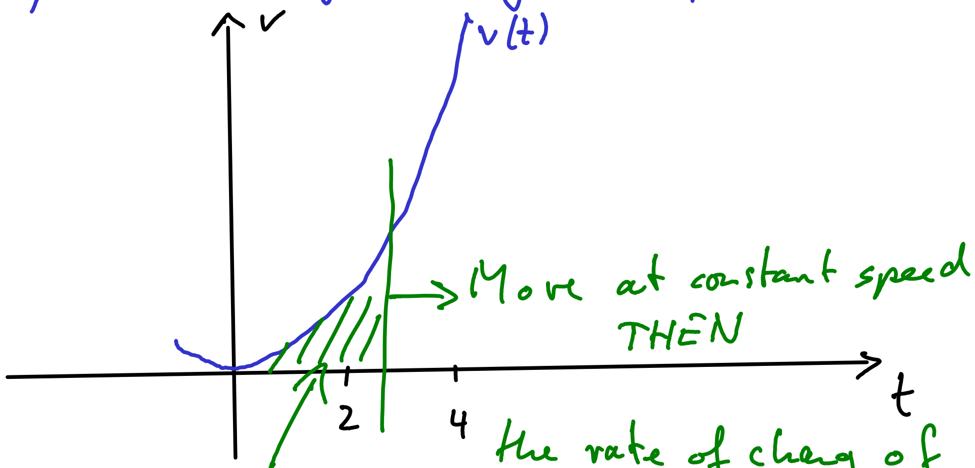
Fixing yesterdays calculations

We were given speed $v(t) = 5t^2$, and asked for average speed between $t=2$ and $t=4$

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time used}}$$

So for a solution, we need to get the distance travelled between $t=2$ and $t=4$!

velocity = rate of change of position



the rate of chang of this area is $v(t)$!

Solution: let $x(t)$ be the position of the skydiver. Then the rate of change of $x(t)$, that is $\frac{dx}{dt}(t)$ equals $v(t)$. In other words

$$x(t) = \int v(t)\,dt = \int 5t^2\,dt = \frac{5}{3}t^3 + c,$$

$c = \text{const.}$

$$\text{average speed} = \frac{x(4) - x(2)}{2} = \frac{5}{6}(4^3 - 2^3) = \frac{5}{3} \times 28 \approx \underline{\underline{46.67}}$$

Secondly, we were asked the rate of change of $v(t)$ at time $t=3$. We did that one fine.

Limits in more detail

Limits are taken of sequences of numbers, but not all sequences have limits

Example: Consider $f(x) = \frac{x^2-1}{x-1}$. This function is not defined at $x=1$. But we can ask what happens when x gets close to 1. We write

$\lim_{x\to 1} f(x)$ to indicate that we let x approach 1 but never $x=1$.

Here we can do trick:

$$\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} \overset{x\neq 1}{=} x+1 \xrightarrow[x\to 1]{} 2$$

which says

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$$

Example: let $f(x) = \frac{x+1}{x^2-1}$, which is not defined at -1 and 1. So what are

$\lim_{x\to 1} f(x)$ and $\lim_{x\to -1} f(x)$??

Again, $\frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} \overset{x\neq -1}{=} \frac{1}{x-1}$ $\xrightarrow{x\to -1} -\frac{1}{2}$, $\xrightarrow[x\to 1]{}$ does not exist