Sequent Rules

Below capital Greek letters denote *finite* sequences of statements, while capital Roman letters stand for statements. A sequent is of the form $\Gamma \Rightarrow \Delta$ and purely formal. However, it helps to interpret it as:

> ***If*** *a choice of truth values makes* *all* *statements in* Γ *true,* ***then*** *it makes* *at least one* *of the statements in* Δ *true.*

or equivalently:

> *For every choice of truth values* *at least one* *of statements in* Γ *is false* **or** *at least one* *of the statements in* Δ *is true.*

$$\underline{\mathbf{R}\wedge}: \quad \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2 \Rightarrow B, \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow A \wedge B, \Delta_1, \Delta_2} \qquad \underline{\mathbf{L}\wedge}: \quad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta} \qquad \frac{\Gamma, B \Rightarrow \Delta}{\Gamma, A \wedge B \Rightarrow \Delta}$$

$$\underline{\mathbf{L}\vee}: \quad \frac{\Gamma_1, A \Rightarrow \Delta_1 \qquad \Gamma_2, B \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2, A \vee B \Rightarrow \Delta_1, \Delta_2} \qquad \underline{\mathbf{R}\vee}: \quad \frac{\Gamma \Rightarrow A, \Delta}{\Gamma \Rightarrow A \vee B, \Delta} \qquad \frac{\Gamma \Rightarrow B, \Delta}{\Gamma \Rightarrow A \vee B, \Delta}$$

$$\underline{\mathbf{L}\rightarrow}: \quad \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2, B \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2, A \rightarrow B \Rightarrow \Delta_1, \Delta_2} \qquad \underline{\mathbf{R}\rightarrow}: \quad \frac{\Gamma, A \Rightarrow B, \Delta}{\Gamma \Rightarrow A \rightarrow B, \Delta}$$

$$\underline{\mathbf{L}\neg}: \quad \frac{\Gamma \Rightarrow A, \Delta}{\Gamma, \neg A \Rightarrow \Delta} \qquad \underline{\mathbf{R}\neg}: \quad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma \Rightarrow \neg A, \Delta} \qquad \underline{\mathbf{LT}}: \quad \frac{\Gamma \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta} \qquad \underline{\mathbf{RT}}: \quad \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow A, \Delta}$$

$$\underline{\mathbf{LC}}: \quad \frac{\Gamma, A, A \Rightarrow \Delta}{\Gamma, A \Rightarrow \Delta} \qquad \underline{\mathbf{RC}}: \quad \frac{\Gamma \Rightarrow A, A, \Delta}{\Gamma \Rightarrow A, \Delta} \qquad \underline{\mathbf{LR}}: \quad \frac{\Gamma, A, B \Rightarrow \Delta}{\Gamma, B, A \Rightarrow \Delta} \qquad \underline{\mathbf{RR}}: \quad \frac{\Gamma \Rightarrow A, B, \Delta}{\Gamma \Rightarrow B, A, \Delta}$$

$$\underline{\mathbf{Cut}}: \quad \frac{\Gamma_1 \Rightarrow A, \Delta_1 \qquad \Gamma_2, A \Rightarrow \Delta_2}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_1, \Delta_2} \qquad \underline{\mathbf{Id}}: \quad A \Rightarrow A$$

Here is an example, how to prove $\Rightarrow \neg(\neg A \vee B) \rightarrow (A \vee B)$, using the sequent rules.

$$\begin{array}{cl}
\underline{A \Rightarrow A} & \mathbf{Id} \\
\underline{\Rightarrow \neg A, A} & \mathbf{R}\neg \\
\underline{\Rightarrow \neg A \vee B, A} & \mathbf{R}\vee \\
\underline{\neg(\neg A \vee B) \Rightarrow A} & \mathbf{L}\neg \\
\underline{\neg(\neg A \vee B) \Rightarrow A \vee B} & \mathbf{R}\vee \\
\Rightarrow \neg(\neg A \vee B) \rightarrow (A \vee B) & \mathbf{R}\rightarrow
\end{array}$$

HEURISTIC: Work from the conclusion and see how a non-atomic statement could be falsified. Put the constituent that needs to be true on the left and the constituent that needs to be false on the right side of the $\Rightarrow$.