CS3304 Logic – Assignment 3

November 11, 2016, Lecturer: Claas Röver

Hand in your solution at the beginning of the lecture on **Friday, 18 Nov 2016**.

QUESTION 1. (a) Convert the following formulæ into conjunctive normal form.

(i) $(p \vee (q \to r)) \to \neg(p \vee \neg q)$ (ii) $q \vee (r \wedge \neg(p \to t))$

(b) Use the Putnam-Davis algorithm to decide the satisfiability of each of the sets of clauses S_1 and S_2.

$$S_1 = \{\{a, b, \neg c, \neg d\}, \{a, \neg b, \neg c, d\}, \{\neg a, b, \neg c, d\}, \{\neg a, \neg b, c, d\}, \{a, f, g\}, \{b, \neg f, g\}, \{c, f, g\}, \{d, \neg f, g\}\}$$

$$S_2 = \{\{p, \neg q, r, s\}, \{\neg p, r, \neg t\}, \{q, t, s\}, \{s, \neg r, p\}, \{\neg s, r\}, \{q, \neg t, p\}, \{t, \neg q\}\}$$

QUESTION 2. Consider the predicate logic $\mathcal{L} = \{(,), , , \forall, \exists, \neg, \wedge, \to, c, E, D, s, f, x, y, z\}$, where c is a constant, x, y and z are variables, E and D predicate letters and s and f function letters. Let $\mathcal{I}$ be the interpretation with domain $D_{\mathcal{I}} = \mathbb{N}$, the natural numbers including zero, with $\mathcal{I}(c) = 0$, $\mathcal{I}(E)$ the property "is even", $\mathcal{I}(D)(n, m)$ if and only if n divides m, $\mathcal{I}(s)$ the successor function and $\mathcal{I}(f)$ the square function, i.e. $\mathcal{I}(s)(n) = n + 1$ and $\mathcal{I}(f)(n) = n^2$.

(a) Which of the formulæ below are satisfied in the interpretation $\mathcal{I}$?

(b) For each of the formulæ below find an interpretation in which it is satisfied.

(c) For each of the formulæ below find an interpretation in which it is not satisfied.

(i) $(\forall x)(E(x) \to E(f(x)))$ (ii) $(\forall x)E(x) \to D(s(s(c)), s(c))$

(iii) $(\exists x)(E(x) \wedge E(s(c)))$ (iv) $(\forall x)(\exists y)(D(x, y) \wedge E(y))$

QUESTION 3. Decide whether the following arguments are valid, by translating into predicate logic and using semantic tableaux.

(a) There are exercises which are difficult to solve but have short answers. Every exercise with a long answer is difficult to solve. This exercise has a short answer and therefore it is easy to solve.

(b) Every exercise in formal logic with a long answer is difficult to solve or an induction. An exercise with a short answer that is easy to solve cannot be an exercise in formal logic. This exercise in formal logic is not an induction. Therefore, it must be difficult to solve.

QUESTION 4. Below A, B and C are well formed formulæ whose free variables are precisely those named in parentheses.

(a) Using semantic tableaux, prove that the following formulæ are valid.

(i) $(\exists x)(A(x) \wedge B(x)) \to ((\exists x)A(x) \wedge (\exists x)B(x))$

(ii) $(\exists x)(\forall y)C(x, y) \to (\forall y)(\exists x)C(x, y)$

(b) Decide, and justify, whether the converses of the above formulæ given below are also valid.

(i) $((\exists x)A(x) \wedge (\exists x)B(x)) \to (\exists x)(A(x) \wedge B(x))$

(ii) $(\forall y)(\exists x)C(x, y) \to (\exists x)(\forall y)C(x, y)$