CS3304 Logic – Assignment 1

September 30, 2016, Lecturer: Claas Röver

Hand in your solution at the beginning of the lecture on **Friday, 7 Oct 2016**.

QUESTION 1. Using truth tables verify the following.

(a) $((A \to B) \land \neg B) \to \neg A)$ is a tautology.

(b) $(A \lor B) \to C \equiv (A \to C) \land (B \to C)$

(c) $((A \land B) \lor (A \land \neg C)) \leftrightarrow (A \land (C \to B))$ is a tautology.

(d) $(A \lor \neg B) \to (A \to B)$ is satisfyable but not a tautology.

QUESTION 2. Using semantic tableaux decide

(a) which of the following sets of statements are consistent.

(i) Pete has red hair and Pete is Irish if Sean is Irish. Sean is Irish if he does not have red hair. If Sean has red hair then either Pete is Irish or Pete does not have red hair but not both.

(ii) Emma lives in Cork or Mary lives in Cork. Nelly does not live in Cork unless Emma lives in Cork. If Emma lives in Cork, then Mary lives in Cork and Nelly does not live in Cork. Not all three girls live in Cork.

(b) whether the following argument is valid.

If Luke has stolen the sword or the ring then he will go to prison. Luke will go to prison unless he hasn't stolen the sword. Luke will not go to prision. Therefore, Luke has not stolen the ring.

QUESTION 3. Find proofs for the following using the natural deduction rules.

(a) $\{A, B \lor C\} \vdash (A \land B) \lor (A \land C)$

(b) $\vdash (A \land B) \lor (A \land C) \to A \land (B \lor C)$

Why is it not correct to say that the two statements above prove one of the distributivity laws? What could be done to make such a proof complete?

QUESTION 4. Below you see four cards on a table. Each card has a letter on one side and an integer on the other. For each of the following claims, say which cards you need to turn over in order to verify the claim. Justify your answers.

(a) A card with a vowel has an odd number on the other side.

(b) A card has an odd number or a vowel on it.

(c) A card with an L has a 6 on the other side.

(d) A card has a vowel if and only if it has an odd number on it.

L	5	6	E