CS3304 Logic – Problem Sheet 1

September 13, 2016, Lecturer: Claas Röver

QUESTION 1. Verify the following equivalences using truth tables.

(a) $(A \wedge B) \vee (A \wedge \neg B) \equiv A$
(b) $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$
(c) $A \vee (A \wedge B) \equiv A$
(d) $A \wedge (A \vee B) \equiv A$

QUESTION 2. Simplify the following propositions.

(a) $A \wedge (\neg A \vee B) \vee B \vee (A \wedge (A \vee B))$
(b) $\neg A \rightarrow \neg(A \rightarrow \neg B)$
(c) $(A \rightarrow B) \rightarrow ((A \rightarrow \neg B) \rightarrow \neg A)$
(d) $(A \rightarrow (A \vee \neg A)) \wedge A$

QUESTION 3. Decide which of the following pairs of propositions are equivalences.

(a) $A \rightarrow (B \rightarrow C)$ and $(A \rightarrow B) \rightarrow C$
(b) $(A \vee \neg B) \rightarrow C$ and $(\neg A \wedge B) \vee C$

QUESTION 4. For each of the following propositions, decide whether it is satisfyable, a tautology or unsatisfyable, using truth tables.

(a) $(A \rightarrow B) \vee (B \rightarrow C)$
(b) $(A \vee B) \rightarrow (C \wedge B \wedge \neg A)$
(c) $(A \wedge B) \rightarrow (\neg A \rightarrow \neg B)$
(d) $A \wedge B \wedge C \wedge (A \rightarrow (\neg B \wedge C))$

QUESTION 5. Using semantic tableaux, decide which of the following sets of propositions are consistent, and for those that are give at least one choice of truth values for the atomic statements satisfying the whole set.

(a) $A \vee B$, $A \rightarrow B$, $\neg B$

(b) $(A \wedge B) \rightarrow C$, $\neg A \rightarrow D$, $B \wedge \neg C \wedge \neg D$

(c) $A \leftrightarrow B$, $B \leftrightarrow C$, $\neg A$, $\neg C$