MA410 Artificial Intelligence Quiz - 4th November 2010.

Attempt *all* questions. Time allowed: *45* mins.

Reference allowed: Unmarked copy of handout *"MA410 Artificial Intelligence - Review of Logic"*

Total marks given per question: 1. *[34 marks]* 2. *[30 marks]* 3. *[19 marks]* 4. *[17 marks]*

Steps taken must be *clearly* indicated for full marks.

1. (a) *[3 marks]* Name *3* proof methods used in propositional logic. (*hint: see 3.3 on logic handout*)
   (b) (i) *[7 marks]* Show that the following axiom is a tautology for all propositions P and Q:

$$\text{Ax3: } (\neg P \to \neg Q) \to (Q \to P)$$

   (ii) *[10 marks]* Use a combination of the axiom above and modus ponens to show that

$$(\neg\neg A \to \neg\neg B) \vdash (A \to B)$$

   (c) *[14 marks]* Prove by contradiction (i.e. method of resolution) that arguments below are valid:
   (a) When it rains, it pours. A storm guarantees rain. Therefore a storm means it pours.
   (b) $(p \to q) \leftarrow r,\ r \therefore q \vee (\neg p \wedge r)$.

2. (a) (i) *[3 marks]* Define the term *sentence* as used in predicate logic.
   (ii) *[7 marks]* Give a true interpretation (stating why it is true) for the sentence

$$\forall x \forall y\ G(x,y) \leftrightarrow L(h(y),h(x)), \quad \textit{(hint: think what might G \& L stand for.)}$$

   for chosen domain $\mathcal{D}$, variables x, y, function h and distinct predicates G, L (i.e. $G \not\equiv L$).
   (b) *[20 marks]* Given H, K and C below with predicates S and T:

$$H\text{: } \forall x,y\ S(x,y) \leftrightarrow T(y,x) \qquad K\text{: } \forall x \exists y\ S(x,y) \qquad C\text{: } \forall x \exists y\ T(y,x),$$

   (i) put H, K and $\neg C$ into clause form; (ii) use method of resolution to show $H, K \therefore C$.

3. (a) (i) *[2 marks]* What is the head and body of the clause "$a \leftarrow b \wedge c$." ?
   (ii) *[10 marks]* Given knowledge base KB of *clause collections* $c_1, ..., c_7$ below, use *either* a top-down *or* bottom-up proof (stating which method) to show query `?-health` is true.

c_1 : `health` $\leftarrow$ `water` $\wedge$ `exercise` $\wedge$ `food.`
c_2 : `exercise` $\leftarrow$ `football` $\wedge$ `run.`
c_3 : `cycle` $\vee$ `swim` $\vee$ `jog` $\rightarrow$ `exercise.`
c_4 : $\neg$`run` $\vee$ `jog.`
c_5 : `run.`
c_6 : `food.`
c_7 : `water.`

   (iii) *[2 marks]* What does it mean to say that `health` is a logical consquence of KB?
   (iv) *[1 mark]* Which of $c_1, ..., c_7$ (if any) is not a clause?
   (b) *[4 marks]* Give a most general unifier of `p(X, Y, a, b, W), p(r(W), c, F, G, t(F,d)).`[†]

4. Consider the following knowledge base[†]:

- `h(k(L), f(R, b(L,e)), c(R)).` ... *(c_1)*
- `h(k(L), f(R, b(L,R)), c(a)).` ... *(c_2)*
- `g(A,B,C)` $\leftarrow$ `h(Y, f(B,C), c(A)).` ... *(c_3)*

   (a) *[10 marks]* Use a top-down derivation including all substitutions for one answer to the query:

`?- g(a, X, b(d,e)).` ... *(c_4)*

   (b) *[5 marks]* Show whether or not there are any more answers.
   (c) *[2 marks]* Indicate which of c_1,...,c_4 are definite, goal and/or horn clauses.

[†]capital letters correspond to variables and lower case letters/words correspond to atoms (as in prolog convention).