MA410 Artificial Intelligence - Fuzzy Logic Problem Sheet

1. (a) What is fuzzy logic? Define the difference between a crisp set and a fuzzy set.
   (b) What is a hedge? How do hedges modify the existing fuzzy sets?

2. (a) Prove DeMorgan's laws for fuzzy logic, i.e. for any two fuzzy sets A and B:
   $$\overline{A \cup B} = \overline{A} \cap \overline{B} \qquad \overline{A \cap B} = \overline{A} \cup \overline{B}$$
   (b) Let X be the reference superset. Determine if for all fuzzy sets $A \in X$:
   $$(i)\ A \cup \overline{A} = X \quad (ii)\ A \cap \overline{A} = \phi$$

3. (a) Given $\mu_A(x) = 0.5$, $\mu_B(x) = 0.2$, $\mu_C(x) = 0.9$ for fuzzy sets A, B and C, find
   (i) $\mu_{\neg A}(x)$ (ii) $\mu_{\neg A \cup B}(x)$ (iii) $\mu_{(\neg A \cup B) \cap C}(x)$ (iv) $\mu_{\neg(B \cup C)}(x)$
   (b) Suppose we are given the t-norm operator $\otimes : (x, y) \mapsto xy$ and the corresponding t-conorm operator $\oplus$, find
   (i) $\mu_{A \otimes C}(x)$ (ii) $\mu_{\neg A \oplus B}(x)$ (iii) $\mu_{(C \cup B) \oplus (C \otimes A)}(x)$ (iv) $\mu_{\neg(B \oplus C)}(x)$
   (c) Calculate (i)-(iv) above expect use the t-conorm operator $\oplus : (x, y) \mapsto \min(1, x + y)$ and the corresponding t-norm $\otimes$.

4. Find $(i)\ \mu_{1/2}(x)$ $(ii)\ \mu_{1/5}(x)$ $(iii)\ \lambda_{9/16}(x)$ (where subscript indicates level of clip)
   $$(iv) \int_0^{10} \max\left(\mu_{1/2}(x), \lambda_{9/16}(x)\right) dx \quad (v) \int_0^{10} \max\left(\mu_{1/2}(x), \lambda_{9/16}(x)\right) x dx,$$
   $$\text{where} \quad \mu(x) = \begin{cases} 1, & 0 \le x \le 2 \\ \frac{6-x}{4}, & 2 < x \le 6 \\ 0, & x > 6 \end{cases}, \quad \lambda(x) = \begin{cases} 0, & 0 \le x \le 3 \\ \frac{(x-3)^2}{16}, & 3 < x \le 7 \\ 1, & x > 7 \end{cases}$$

5. The intoxication level of clients at the college bar is measured in number of pints (on a range $[0, 15]$) consumed and is broken down into classifications *sober*, *tipsy*, *drunk* or *plastered* according to the following membership functions:

   $$\mu_{sober}(x) = \begin{cases} 1, & x \le \frac{1}{2} \\ \frac{5}{2} - x, & \frac{1}{2} < x \le \frac{5}{2} \\ 0, & x > \frac{5}{2} \end{cases} \qquad \mu_{drunk}(x) = \begin{cases} 0, & x \le 2 \\ \frac{1}{4}(x-2)^2, & 2 < x \le 4 \\ 1, & 4 < x \le 6 \\ \frac{1}{2}(8-x), & 6 < x \le 8 \end{cases}$$

   $$\mu_{tipsy}(x) = \begin{cases} x - 1, & 1 \le x \le 2 \\ 3 - x, & 2 < x \le 3 \\ 0, & \text{elsewhere} \end{cases} \qquad \mu_{plastered}(x) = \begin{cases} 0, & x \le 4 \\ \frac{1}{2}(x-4)^{1/2}, & 4 < x \le 8 \\ 1, & x > 8 \end{cases}$$

   (a) Draw a fuzzy graph of intoxication indicating level of membership for classifications *sober*, *tipsy*, *drunk* and *plastered*.
   (b) Find $\lambda(z) = [\mu_{sober}(\frac{z}{2})]^2$ and $\chi(z) = [\mu_{plastered}(2z + 3)]^2$ in terms of z only.
   (c) Find (i) $\mu_{sober}\left(\frac{1}{2}\right)$ (ii) $\mu_{drunk}\left(\frac{7}{2}\right)$ (iii) $\mu_{tipsy}(2)$ (iv) $\mu_{\text{“very plastered”}}(7)$

6. Outline and explain the four steps taken in Mamdani's method for fuzzy inference. What are the differences between Mamdani's and Sugueno's method?

7. Given the fuzzy sets for **height** of *tall* and *short* (measured in metres) described by membership functions:

$$\mu_{tall}(x) = \begin{cases} 0, & \text{if } x < 1.6 \\ (x-1.6)/0.2, & \text{if } 1.6 \leq x < 1.8 \\ 1, & \text{if } x \geq 1.8 \end{cases}$$

$$\mu_{short}(x) = \begin{cases} 1, & \text{if } x < 1.6 \\ (1.8m-x)/0.2, & \text{if } 1.6 \leq x < 1.8 \\ 0, & \text{if } x \geq 1.8 \end{cases}$$

   (a) Sketch the corresponding graphs of these functions.

   (b) Calculate $\mu_{tall \cup short}(x)$ and $\mu_{tall \cap short}(x)$.

   (c) Show that the complement of $\mu_{tall}(x)$ is $\mu_{short}(x)$.

   (d) Given the additional fuzzy sets of **strength** *strong* and *weak* with membership functions:

$$\mu_{strong}(y) = \begin{cases} 0, & \text{if } y < 30 \\ (y-30)/20, & \text{if } 30 \leq y < 50 \\ 1, & \text{if } y \geq 50 \end{cases}$$

$$\mu_{weak}(y) = \begin{cases} 1, & \text{if } y < 30 \\ (50-y)/20, & \text{if } 30 \leq y < 50 \\ 0, & \text{if } y \geq 50 \end{cases}$$

   and the fuzzy rules with output for **weight**

   IF tall OR strong THEN heavy
   IF short AND weak THEN light

   Calculate the membership function for **weight** when

      i. **height** $= 1.65m$, **strength** $= 30kg$

      ii. **height** $= 1.70m$, **strength** $= 45kg$

   (e) Given that $\mu_{heavy}(z) = \mu_{strong}(z/3)$ and $\mu_{light}(z) = \mu_{weak}(\frac{z-10}{2})$,

      i. Find aggregate sets for values in 7(d)i and 7(d)ii.

      ii. Hence find the centre of gravity using definite integration when (i) clipping, (ii) scaling; is used where **weight** is in range $[0, 200]$.

      iii. Use Sugueno's method to calculate the centre of gravity by taking spikes.

8. The fuzzy sets A, B and C are all defined on the universe $X = [0, 5]$ with the following membership functions:

$$\mu_A(x) = \frac{1}{1+5(x-5)^2} \quad \mu_B(x) = 2^{-z} \quad \mu_C(x) = \frac{2x}{x+5}$$

   (a) Sketch the membership functions

   (b) Define the intervals along the x-axis corresponding to the α-cut sets for each of the fuzzy sets for the values:

$$(i)\ \alpha = 0.2 \quad (ii)\ \alpha = 0.4 \quad (iii)\ \alpha = 0.7 \quad (iv)\ \alpha = 0.9$$