## MA410 Artificial Intelligence - Constraint Satisfaction Problems

1. Define the following terms:
   (a) A constraint satisfaction problem
   (b) Model of a CSP
   (c) Domain consistency
   (d) Constraint network
   (e) Arc consistency
2. A solution to an $N$-Queens problem is a placement of $N$ chess queens on an $N \times N$ chessboard so that no queen can attack any other queen. Consider the 4-Queens problem. Denote the queens as variables $q_i$ ($i \in \{1, 2, 3, 4\}$) where $q_i$ lies in column $i$ of the chessboard.
   (a) Draw out the game tree & find one solution using forward checking with backtracking.
   (b) Write down the set of domains for each $q_i$.
   (c) Write down the set of constraints.
   (d) Use arc consistency to find one solution.
3. Repeat question 2 above for the 5-Queens problem.
4. Each sudoku below shows squares which contain either a variable $v_i$ or a no. from 1 to 4. Using the normal rules of Sudoku,
   (a) write out the set of constraints and the constraint graph.
   (b) use arc consistency to find unique solutions for the variables $v_i$.

(i)

| | | | |
|---|---|---|---|
| $v_1$ | 4 | 2 | $v_2$ |
| $v_3$ | 2 | $v_4$ | 4 |
| 4 | $v_5$ | $v_6$ | $v_7$ |
| 2 | $v_8$ | 4 | 1 |

(ii)

| | | | |
|---|---|---|---|
| 1 | 3 | 2 | $v_1$ |
| $v_2$ | $v_3$ | $v_4$ | $v_5$ |
| $v_6$ | $v_7$ | 1 | $v_8$ |
| 2 | $v_9$ | $v_{10}$ | 3 |

(iii)

| | | | |
|---|---|---|---|
| 4 | $v_1$ | 1 | $v_2$ |
| $v_3$ | $v_4$ | 4 | 2 |
| $v_5$ | 4 | $v_6$ | $v_7$ |
| 2 | 1 | $v_8$ | 4 |

5. Each minesweeper below shows squares which contain either
   - a number $n$ (touching exactly $n$ bombs),
   - "B" for a bomb, or
   - a variable name $x_i$ (which has value 1 if it contains a bomb and 0 otherwise).

   (a) write out the set of constraints and the constraint graph.
   (b) use arc consistency to find unique solutions for the variables $x_i$.

(i)

| | | | |
|---|---|---|---|
| $x_1$ | B | $x_2$ | 2 |
| B | 6 | $x_3$ | B |
| 3 | $x_4$ | B | 3 |
| 2 | B | 3 | $x_5$ |

(ii)

| | | | |
|---|---|---|---|
| 2 | $x_1$ | B | 2 |
| B | $x_2$ | 6 | B |
| 3 | B | B | $x_3$ |
| $x_4$ | $x_5$ | 4 | 2 |

(iii)

| | | | |
|---|---|---|---|
| B | $x_1$ | 4 | $x_2$ |
| 2 | $x_3$ | B | B |
| $x_4$ | 4 | $x_5$ | 3 |
| B | B | 2 | $x_6$ |

6. What code and methods in prolog could we use to
   (a) represent the data in; and (b) solve
   the above problems.