MA410 Artificial Intelligence - Problem Sheet 4 - SLD Resolution

1. Consider the following knowledge base

   - `ap(emp,L,L).`
   - `ap(c(H,T),L,c(H,R))` $\leftarrow$ `ap(T,L,R).`
   - `adj(A,B,L)` $\leftarrow$ `ap(F,c(A,c(B,E)),L).`

   (a) Use SLD resolution (top-down derivation) including all substitutions for one answer to the query: `?- adj(b,Y,c(a,r)).`

   (b) Explain whether or not there are any other answers. If so, continue the derivation showing another example.

2. Consider the following Horn argument of propositional logic and its SLD search space:

   (1) $R \wedge S \rightarrow P$
   (2) P
   (3) S
   (4) $S \rightarrow Q$

   $P \wedge Q$

   (a) Display an SLD resolution proof for this argument. Be sure to show the premises that get used in the proof.

   (b) Use a forward-proof (i.e. bottom-up, starting with premises) and derive new formulae until the conclusion is reached. Give the forward proof that corresponds to the previous SLD resolution proof.

   (c) What does the SLD proof in (a) tell us about the validity of the argument? Justify your answer, citing the relevant theorem.

3. Consider the following Horn clause argument:

   (1) $A_1 \rightarrow A$ (2) $A_2 \rightarrow A$
   (3) A_1 (4) A_2
   (5) $B_1 \rightarrow B$ (6) $B_2 \rightarrow B$
   (7) $B_3 \rightarrow B$ (8) B_1

   $A \wedge B$

   Draw the entire SLD search space for this argument using the (i) leftmost selection function, (ii) rightmost selection function.

4. Let $\mathcal{P}$ be the following set of definite clauses:

   p1. $P(x, f(x))$ (Your father is one of your parents.)
   p2. $P(x, m(x))$ (Your mother is one of your parents.)
   p3. $P(x, y) \rightarrow A(x, y)$ (Your parents are ancestors of yours.)
   p4. $P(x, y) \wedge A(y, z) \rightarrow A(x, z)$ (Your parents' ancestors are ancestors of yours.)

   (a) Using the leftmost selection function, give the SLD search space to depth 3 for the argument whose premises are P and whose conclusion is $A(x, j)$. Explain what the rest of the search space looks like. Does it contain any answers?

   (b) Repeat the previous question, except this time replace premise p4 with p4'. $P(y, z) \wedge A(x, y) \rightarrow A(x, z)$ (Your ancestors' parents are ancestors of yours.)

   (c) Using the leftmost selection function, give the SLD search space to depth 4 for the argument whose premises are $\mathcal{P}$ and whose conclusion is $A(j, x)$?

5. Prove that the SLD inference rule for first-order logic is sound.