MA410 Artificial Intelligence - Problem Sheet 2 - Predicate Logic

1. Define the terms (i) *term*, (ii) *predicate*, (iii) *well-formed formula*, (iv) *sentence* as used in predicate logic. Give an example of each. What is an *interpretation* of a sentence? When is the argument $H_1, H_2, \ldots, H_k \therefore C$ valid?

2. Specify a model pair $\langle \mathcal{D}, \phi \rangle$ (where $\mathcal{D}$ is the domain and ϕ a function acting on symbols) that is true in one case and false in the other for the sentences:

   (a) $(\forall x \exists y P(x,y)) \rightarrow (\exists y \forall x P(x,y))$ (b) $\exists x\, \forall y P(x, g(y))$ (c) $\forall x, y\, [G(x,y) \leftrightarrow L(y, g(x))]$

   using symbols x and y which are variables, $g(.)$ a function and predicates P,G & L.

3. Given the sentences "$H_1 : \forall x \exists y R(x,y)$" and "$C : \exists x, y R(x,y)$", convert H_1 and $\neg C$ into clause form and use resolution to show that C follows logically from H_1.

4. Let $A(x,y), P(x,y)$, $S(x,y)$ be interpreted as x is an aunt, parent, sister of y respectively.

   (a) Write sentences H_1, H_2, H_3 and C in predicate calculus where

   H_1 defines A in terms of P and S
   H_2 says everyone has a parent
   H_3 says everyone has a sister
   C says everyone has an aunt

   (b) Convert each of H_1, H_2, H_3 and $\neg C$ to clause form.

   (c) Use resolution to show that C follows logically from H_1, H_2, H_3.

5. Given the following predicates:
   $On(x,y) =$ "x is on y", $B(z) =$ "z is a brick", $g =$ "the ground".

   (a) Write sentences H, K and C in predicate calculus to say:
   H: every brick is on the ground or on another brick,
   K: no brick is on a brick which is also on a brick,
   C: if one brick is on another, the second one is on the ground.

   (b) Convert H, K and $\neg C$ to clause form.

   (c) Use resolution to show that C is a logical consequence of H and K.

6. Show whether each of the following claims hold:

   (a) $\forall x Q(x) \models \exists x Q(x)$. (b) $\forall x P(x) \rightarrow Q(x) \models \exists x P(x) \vee Q(x)$.

7. Consider the first-order predicate calculus with the following lexicon:

   Predicate symbols: P (with arity[1] 2), and Q (with arity 1);
   Function symbols: f (with arity 2) and a (with arity 0);
   Variables: x, y and z.

   In each of the following formulae, draw lines to indicate which quantifer (if any) binds every occurrence of every variable. Indicate which are sentences. (Eg. In $\forall x(P(x) \rightarrow \forall x \forall y Q(x,y,x))$ you would connect the 1st occurrence of x to the 2nd occurrence of x, and you would connect the 3rd, 4th and 5th occurrences of x. You would also connect the two occurrences of y.)

   (a) $\forall x P(x,x)$
   (b) $\exists x \forall x (Q(x) \rightarrow Q(x))$
   (c) $\forall x (Q(x) \rightarrow \forall x Q(x))$
   (d) $\forall x((Q(x) \rightarrow \forall x Q(x)) \rightarrow Q(x))$
   (e) $\forall x((\exists y Q(x)) \rightarrow Q(y))$
   (f) $\exists x \neg P(f(x,y), a)$

[1]the number of arguments that the function takes