MA410 Artificial Intelligence - Problem Sheet 1 - Propositional Logic/Calculus

1. (i) What is an axiom? (ii) What is a well-formed formula?

   (iii) What proof method could be used to show that the boolean algebra axioms (see logic handout) hold for propositional calculus?

   Use this method to show that the absorption rule holds for atomic propostions p, q, i.e.

   $$p \vee (p \wedge q) \equiv p \quad \text{and} \quad p \wedge (p \vee q) \equiv p.$$

   (iv) Why does this imply the absorption rule is applicable for all well-formed formulae? Show similarly to above that

   $$(p \to q) \vee ((p \to q) \wedge r) \equiv (p \to q).$$

2. Find the conjunctive normal form (CNF) of (i) P and (ii) $\neg P$, where P is the statement form (WFF): (a) $p \wedge (\neg q \vee r)$ (b) $r \to (q \vee p)$ (c) $\neg((p \to \neg q) \to r)$

3. Determine by (propositional resolution) whether or not the following set of clauses is inconsistent:

   $$\{\{p_1, \neg p_2, p_3, \neg p_4\}, \{p_1, \neg p_3\}, \{\neg p_1, p_2, \neg p_4\}, \{p_4\}\}$$

4. What does the argument $P, Q \therefore C$ is valid mean? What does it mean to say a proof method is refutation complete? Determine whether the following arguments are valid or not by using the method of (propositional) resolution outlining the steps carefully:

   (a) $(p \wedge q) \vee \neg r,\ p \to (r \wedge \neg q) \ \therefore \neg r$
   (b) $p, p \to q, q \to r \therefore r$
   (c) $\neg p \vee (q \wedge r), q \to (p \wedge \neg r) \therefore \neg p$
   (d) $p \vee q \vee r \therefore p \wedge q \wedge \neg s$
   (e) $p \to (q \to r), q \therefore p \to r$
   (f) $\neg p \wedge (\neg q \vee r), q \wedge r, r \to s, s \vee \neg p \therefore \neg(\neg s \wedge p)$

5. Write the following in terms of propositional calculus and determine whether the statements are valid using the method of resolution:

   (a) When it rains, it is wet. It is dry. Therefore it did not rain.
   (b) The server breaks down. The server breaks down when there is too much traffic. Therefore there is too much traffic.
   (c) If the robot is switched on and it reverses then it has hit an obstacle.
   The robot will go North or South.
   There are no obstacles North.
   There is an obstacle South.
   The robot is switched on and it goes North.
   Therefore it does not reverse.

6. A truth valuation is a function from a set of wff to a set $\{0, 1\}$ s.t. for any wff A and B,

   $$v(\neg A) \neq v(A), \quad v(A \to B) = 0 \ \textit{iff} \ v(A) = 1 \text{ and } v(B) = 0.$$

   Show using this definition that each of the following is a tautology:
   (a) $A \to A$ (b) $\neg\neg A \to A$ (c) $(\neg A \to A) \to A$

7. Prove the following results using Axiomatic Propositional Logic:

   (a) $(\neg\neg A \to \neg\neg B) \vdash (A \to B)$
   (b) $\vdash (\neg\neg A \to \neg\neg B) \to (A \to B)$
   (c) $\{B \to \neg\neg B, \neg\neg A \to A, (A \to B)\} \vdash (\neg B \to \neg A)$
   (d) $A \vdash C \to (A \to (B \to A))$.