

MA410 Artificial Intelligence

Prolog Practical Session I on N-Queens and Sudoku

N-Queens Problem

A solution to an N -Queens problem is a placement of N chess queens on an $N \times N$ chessboard so that no queen can attack any other queen. A queen is “safe” (cannot be attacked) if no other queen is on the same row, on the same column, or on the same diagonal. The following figure shows a solution to the 8-Queens problem: all 8 queens are “safe”:

					Q		
		Q					
				Q			
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Q							
			Q				
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							Q

1. Enter in two solutions to the 4-Queens problem in the grids below:

2. What process did you use in order to find the solutions?

3. Consider position $(1,1)$ on an $N \times N$ board (where position is of the form: $(row, column)$). What positions on the board attack $(1,1)$?

- We know that a queen cannot be on the same row, hence we know that everything of the form $(1, Col)$, $Col \in [2, \dots, N]$ will attack the queen.
- We know that a queen cannot be on the same column, hence we know that everything of the form $(Row, 1)$, $Row \in [2, \dots, N]$ will attack the queen.
- We know that a queen cannot be on the same diagonal, so for instance $(2, 2)$, $(3, 3)$ etc. would also attack the queen. etc.

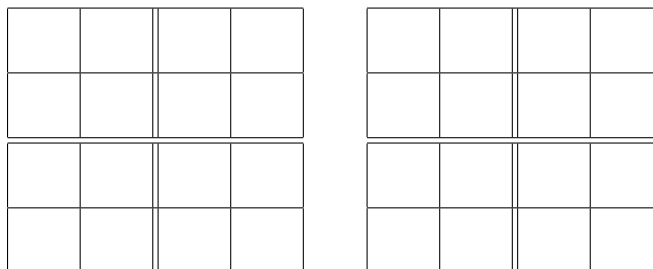
Sudoku Problem

A Sudoku puzzle of size M is a square grid with M^2 rows and M^2 columns of cells divided up into M^2 $M \times M$ sub-squares, and an alphabet of M^2 unique symbols, $\{a_1, a_2, \dots, a_{M^2}\}$. Usually $a_i = i$ are integers. A solution to a Sudoku puzzle is a placement of M^4 alphabet symbols into the grid cells such that the following constraints are respected:

- (a) no row can contain more than one instance of a symbol
- (b) no column can contain more than one instance of a symbol
- (c) no $M \times M$ sub-square can contain more than one instance of a symbol

We will say that a symbol placed on the grid at position (r_1, c_1) is attacking a symbol at position (r_2, c_2) (and vice-versa) if the constraints are not respected.

1. Enter two distinct arrangements of symbols into the 4×4 Sudoku grids using symbols $\{1, 2, 3, 4\}$ by entering each symbol individually at a time (i.e. start by placing all the 1's, then 2's, 3's and finally 4's):



2. How did you ensure that individual symbols didn't attack each other?

3. Given a specific position in the form (row,column) say (r, c) on an $N \times N$ board, where a specific symbol is placed at (r, c) , what values must (X, Y) satisfy in order that:
(Hint : You will need to use the operator $\lfloor \rfloor$ when comparing positions in subblocks).
 - (a) (r, c) and (X, Y) are attacking each other.

 - (b) (X, Y) is placed in a legal position (i.e. does not attack (r, c) and is not equal to (r, c))

4. We can see that the idea is similar to the N-Queens problem except that attacks are specified in a different way. In fact, we need to repeat for each of the symbols, i.e. first place all 1's that don't attack and then all 2's that don't attack each other and whose positions can't overlap with the positions chosen for the 1's etc.

- (a) We will again enter in values on a blank 4×4 board but in an algorithmic way.
- i. First we write out all the legal positions for a 4×4 board. These are put into the four grids below which we have labelled 1, ..., 4.
 - ii. Start with symbol 1 and place it somewhere in the first column in one of the empty grids below.
 - Whichever position you chose, circle this position on all grids 1, ..., 4.
 - Now 'X' out all positions (only) on grid 1 that are attacked by this symbol.
 - iii. Repeat the process for the 3 remaining columns.
 - iv. Now repeat the steps above for the remaining symbols 2, 3 and 4 until you have completed the grid.

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5. In an actual Sudoku board, we are given initial positions that are occupied. How would we delete these from our lists and also restrict attack squares? Use the grids below and one of the empty grids above to put in some initial values and repeat a similar process as above.

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6. At next week's lab, we will look at coding the N-Queen's and Sudoku problems in prolog.