

MA410 Artificial Intelligence - CSP Definitions

Constraint Satisfaction Problem

Defined as a triple $\langle X, D, C \rangle$:

- X set of variables
- D domain of values
- C Constraints of form $\langle t, R \rangle$ where
 - t = tuples of variables
 - R = tuples of relations.

Model of CSP

A *model* of a CSP is an assignment of values to variables that satisfy all the constraints.

Domain Consistency

A variable is *domain consistent* if no value of the domain of the node is ruled impossible by any of the constraints.

Constraint Network

A *constraint network* is a graph with

- one node for each variable
- one node for each constraint

Unidirected edges running between variable nodes and constraint nodes whenever a given variable is involved in a given constraint.

Arc Consistency

An arc $\langle A, r(A, B) \rangle$ is *arc consistent* if for each value of $X \in \text{dom}(A)$, there is some value $Y \in \text{dom}(B)$ such that $r(X, Y)$ is satisfied.

Arc Consistency Algorithm

procedure AC(V, dom, R)

Inputs

V : a set of variables

dom : a function such that $dom(X)$ is the domain of var X

R : set of relations to be satisfied

Output

arc consistent domains for each variable

Local

D_X is a set of values for each variable X

for each variable X **do**

$D_X \leftarrow dom(X)$

end for each

$TDA \leftarrow \{\langle X, r \rangle \mid r \in R \text{ is a constraint that involves } X\}$

while $TDA \neq \{\}$ **do**

select $\langle X, r \rangle \in TDA$;

$TDA \leftarrow TDA \setminus \{\langle X, r \rangle\}$;

$ND_X \leftarrow \{x \mid x \in D_X, \exists y \in D_y \text{ s.t. } r(x, y)\}$;

if $ND_X \neq D_X$ **then**

$TDA \leftarrow TDA \cup \{\langle Z, r' \rangle \mid r' \neq r \text{ and } r \text{ involves } X, Z \neq X\}$;

$D_X \leftarrow ND_X$;

end if

end while

return $\{D_X \mid X \text{ is a variable}\}$

end procedure