MA410 Artificial Intelligence

Assignment 4:
Reasoning with Uncertainty + CSPs

Due: Thursday, 3rd March 2011
by 16:00hrs

Name(s):	
Student ID(s):	

Answer all questions fully within this booklet.
No loose pages will be accepted.

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1. (a) What is uncertainty? _____

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(b) Name *three* ways in which knowledge may be uncertain and give examples for each.

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(c) Name *two* methods that Artificial Intelligence uses in order to deal with uncertainty.

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2. (a) Let the probabilities that "the referee is sick" and rain (assuming independence) be 0.2 and 0.5 resp. If there is rain, the chance that it is heavy is 0.5. Suppose the rule

"if rain is heavy or the referee is sick then no football"

and otherwise there is a 90% chance of football. What is the probability of football?

(b) Suppose a computer can be infected with a virus. If it has a virus, it has either a worm, a trojan or both (assume they are independent). An anti-virus program is used.

Events are given by I: *computer is infected*, W: *computer has worm*, T: *computer has trojan*, A: *anti-virus program works.*

together with probabilities $P(I) = 0.1,\ P(W \cap \overline{T} \mid I) = 0.25,\ P(T \cap \overline{W} \mid I) = 0.6,$ $P(A \mid W \cap \overline{T}) = 0.8,\ P(A \mid T \cap \overline{W}) = 0.75.$

i. Find $P(W \cap T \mid I)$.

ii. Find $P(A \mid W \cap T)$.

(c) Draw a Bayesian Network Diagram with states *Infected*, *Worm Only*, *Trojan Only*, *Faultless*.

(d) If the computer was fixed by the program, find the chance it had
(i) a worm only, (ii) a trojan only, (iii) both a trojan and worm.

Use Bayes' Formula: $P(A_i \mid B) = \dfrac{P(A_i)P(B \mid A_i)}{\sum_{k=1}^{n} P(A_k)P(B \mid A_k)}$

3. Let $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{C}$ be fuzzy sets and $\mu_{\underset{\sim}{A}}(x) = 0.65$, $\mu_{\underset{\sim}{B}}(x) = 0.7$, $\mu_{\underset{\sim}{C}}(x) = 0.55$.
   Given the t-conorm operator $\oplus : (t, s) \rightarrow \min(1, t + s)$ and corresponding dual t-norm $\otimes$,
   find (i) $\mu_{\underset{\sim}{A}\oplus\underset{\sim}{C}}(x)$ (ii) $\mu_{(\underset{\sim}{A}\cup\underset{\sim}{B})\otimes\overline{\underset{\sim}{C}}}(x)$ and (iii) $\mu_{\overline{\underset{\sim}{A}}\oplus(\underset{\sim}{B}\cap\underset{\sim}{C})}(x)$?

4. Consider the fuzzy logic system given by categories:

   Speed (*slow*, *med* or *fast*) measured by km/hr in range [0,150].
   Weather (*bad* or *good*) measured in range [0,10].
   Gear (*low* or *high*) measured in range [0,6].

   with fuzzy membership functions μ for subsets given in the table below:

Speed

$$\mu_{slow}(x) = \begin{cases} 1, & x \leq 10 \\ \frac{30-x}{20}, & 10 < x \leq 30 \\ 0, & x > 30 \end{cases}$$

$$\mu_{med}(x) = \begin{cases} \frac{x-20}{20}, & 20 \leq x \leq 40 \\ \frac{80-x}{40}, & 40 < x \leq 80 \\ 0, & \text{elsewhere} \end{cases}$$

$$\mu_{fast}(x) = \begin{cases} 0, & x \leq 50 \\ \frac{x-50}{50}, & 50 < x \leq 100 \\ 1, & x > 100 \end{cases}$$

Weather

$$\mu_{bad}(y) = \begin{cases} 1, & y \leq 2 \\ \frac{1}{25}(7-y)^2, & 2 < y \leq 7 \\ 0, & y > 7 \end{cases}$$

$$\mu_{good}(y) = \begin{cases} 0, & y \leq 4 \\ \frac{1}{2}(y-4)^{1/2}, & 4 < y \leq 8 \\ 1, & y > 8 \end{cases}$$

Gear

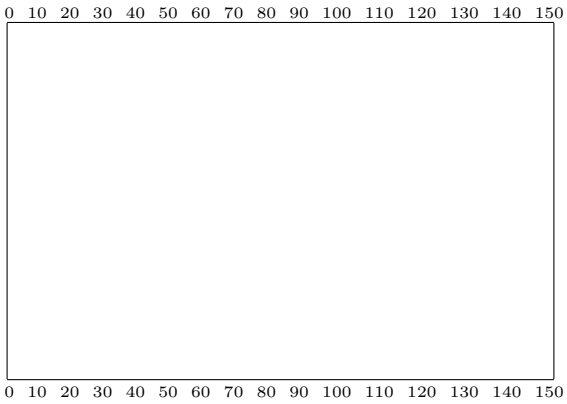
$$\mu_{low}(z) = \mu_{slow}(10z), \quad \mu_{high}(z) = \left[\mu_{good}(2z)\right]^2$$

   (a) Find $\mu_{low}(z)$ and $\mu_{high}(z)$ in terms of z only.

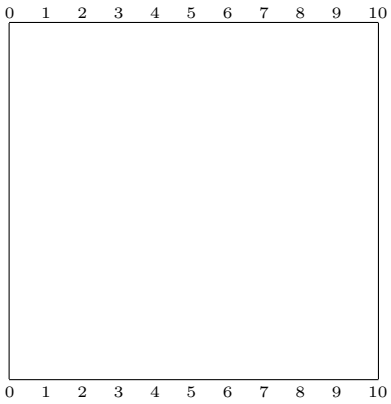
$$\mu_{low}(z) = \begin{cases} \\ \\ \end{cases} \qquad \mu_{high}(z) = \begin{cases} \\ \\ \end{cases}$$

(b) Draw fuzzy graphs for the sets **Speed**, **Weather** and **Gear**.

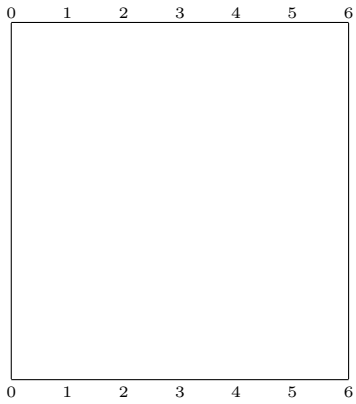
Speed



Weather



Gear



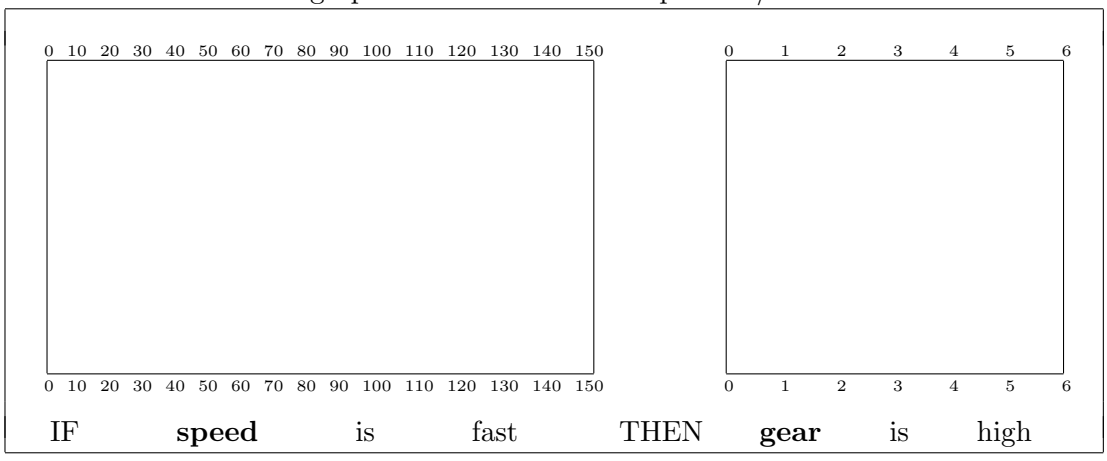
(c) Name and explain briefly the four steps in Mamdani's method for fuzzy inference.

(d) The following fuzzy rules are given:

R1: *IF* ***speed*** *IS fast THEN* ***gear*** *IS high*
R2: *IF* ***speed*** *IS slow THEN* ***gear*** *IS low*
R3: *IF* ***speed*** *IS med AND* ***weather*** *IS bad THEN* ***gear*** *IS high*
R4: *IF* ***speed*** *IS med AND* ***weather*** *IS good THEN* ***gear*** *IS low*

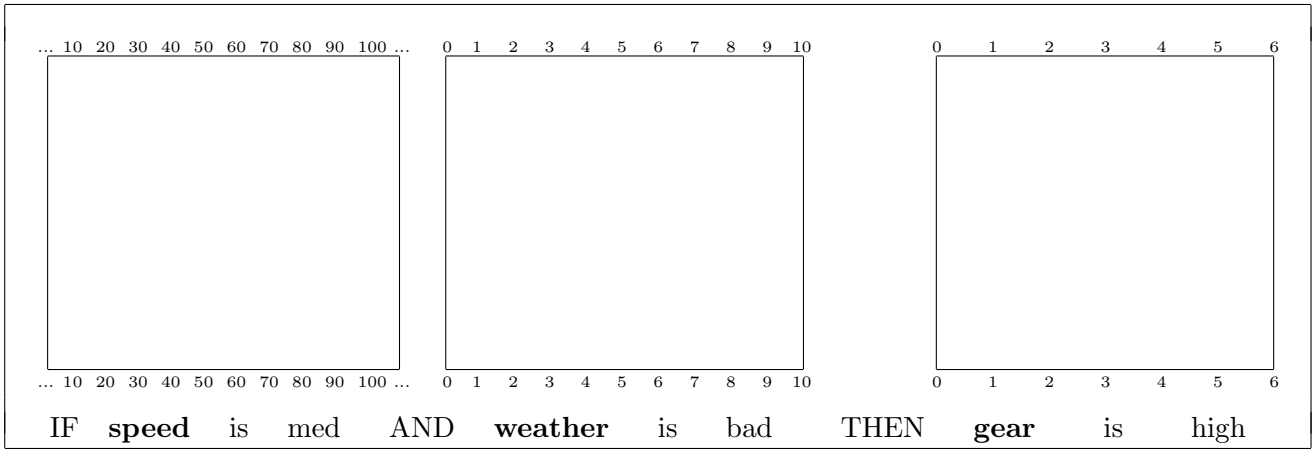
Given inputs Speed = 75 and Weather = 6,

i. Draw rule evaluation graph for **R1** and show output is 1/2.

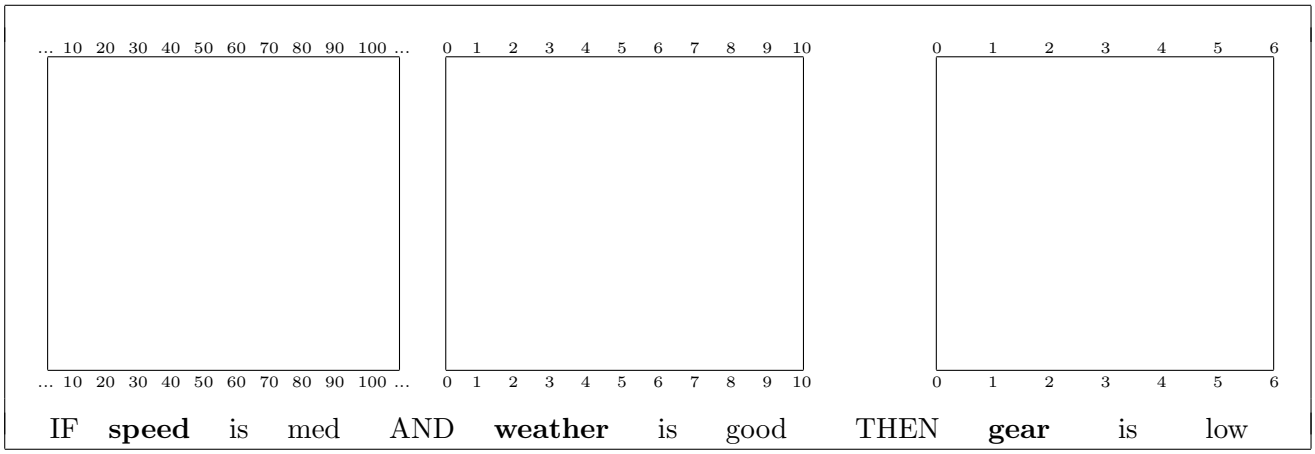


p.t.o.

ii. Draw rule evaluation graph for **R3** and show output is 1/25.



iii. Draw rule evaluation graph for **R4** and show output is 1/8.



(e) Complete Mamdani's method to calculate the centre of gravity using clipping & definite integration.

-1- Aggregation of Rule Outputs: add graphs together above from rule evaluation step.

-2- Calculate the centre of gravity using formula: $\frac{\int_a^b z f(z) dz}{\int_a^b f(z) dz}$

5. (a) Define what is meant by a constraint satisfaction problem (CSP).

(b) The sudoku below shows squares which contain either a variable v_i or a no. from 1 to 4. Write down the set of constraints and the constraint graph.

v_1	3	1	v_2
v_3	1	v_4	3
3	v_5	v_6	v_7
1	v_8	3	4

Set of constraints:

Constraint graph:

(c) Using normal Sudoku rules with arc consistency, find carefully values for v_1, ..., v_8.