22nd Galway Topology Colloquium

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Book of Abstracts

Daron Anderson

Non-Block Points in Hereditarily Decomposable Continua

The celebrated Whyburn non-cutpoint theorem states that every continuum − metric or otherwise − has two or more non-cutpoints. Ten years ago the result was strengthened for metric continua to show the existence of non-block points. These are a special type of non-cutpoints such that the complement of each point contains a dense continuumwise-connected subset. The question is open whether such points are guaranteed to exist for non-metric continua. However the answer is affirmative for continua that are either irreducible, aposyndetic, or separable. In this work we add another special condition to the list, by showing that hereditarily decomposable continua have non-block points.

Paul Bankston

Marquette University, USA

Betweenness and Equidistance in Hyperspaces

We explore what it means when one compact set lies between—or is equidistant from—two others, in the context of metric spaces. We are also interested in notions of convexity that arise from these considerations.

Christopher Boyd

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University College Dublin

Order Continuous Polynomials

A Banach lattice is a vector space which possesses both a topological (norm) structure and a lattice structure. Each of these two structures gives rise to a notion of convergence. In the topological setting it is convergence with respect to the norm, in the lattice setting it is order convergence. In this talk we will explore continuity of polynomials in both of these settings and see that norm and order continuity can give rise to radically different behaviour.

This is joint work with R. Ryan and N. Snigireva.

Klaas Pieter Hart
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Many subalgebras of $\mathcal{P}(\omega)/\mathit{fin}$

A tale of mass murder and mayhem

In answer to a question on Mathoverflow we show that the Boolean algebra $\mathcal{P}(\omega)/\mathit{fin}$ contains a family $\{\mathcal{B}_X : X \subseteq \mathfrak{c}\}$ of subalgebras with the property that $X \subseteq Y$ implies $\mathcal{B}_Y$ is a subalgebra of $\mathcal{B}_X$ and if $X \nsubseteq Y$ then $\mathcal{B}_Y$ is not embeddable into $\mathcal{B}_X$. The proof proceeds by Stone duality and the construction of a suitable family of separable zero-dimensional compact spaces.

Robin Knight
University of Oxford

TBA

TBA

John Mayer
University of Alabama at Birmingham

Complex Dynamics: Polynomials, Julia Sets, Parameter Spaces, and Laminations

Laminations are a combinatorial and topological way to study connected Julia sets of polynomials. While each locally connected Julia set has a corresponding lamination, laminations also give information about the structure of the parameter space of degree $d \geq 2$ polynomials with connected Julia sets. A d-invariant lamination of the unit disc consists of a closed collection of chords, called leaves, which meet at most at their endpoints, and which is forward and backward invariant under the angle-d-tupling map on the unit circle. Of particular interest are leaves in a lamination which are periodic, return for the first time by the identity, and whose endpoints are in different orbits. Such leaves play an important and understood role in the parameter space of quadratic polynomials and in the parameter spaces of unicritical higher degree polynomials, but more study is needed in the more general case of multiple criticality. Here we focus on the first case where there are open questions about the laminations: the angle-tripling map corresponding to degree 3 polynomials with connected Julia set.
Coauthors: Brittany E. Burdette and Thomas C. Sirna

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A few points in pointfree topology

It has long been known from a study of the early papers in topology that some important insights into the study of a topological space can be gleaned by studying such spaces from a lattice-theoretic point of view. The open sets of any topological space form a lattice of a special kind known as a frame; therefore, these objects have risen to some importance as far as their study is concerned. This is the base of pointfree topology. Incidentally, some categorical tools and constructions prove to be important when one does topology in this way. The talk aims to illustrate the importance of pointfree topology via the exhibition of different constructions, giving concrete examples where necessary.

Dowker, C. H, and Papert D, *On quotient frames and subspaces*, Proc. London.Math. Soc. **16** (1966), 275–296.
Isbell J. R, *Atomless parts of spaces*, Math. Scand. **31** (1972), 5–32.
Johnstone, P. T, *Stone Spaces*, Cambridge University Press (1982), Cambridge.
Mthethwa, S. S, *On J-frames,* Topology Appl. **342** (2024), 108772.
Picado J, and Pultr A, *Frames and Locales: topology without points*, Frontiers in Mathematics, Springer, Basel, 2012.
Picado J, and Pultr A, *Separation in Point-Free Topology*, Birkhäuser, Switzerland, 2021.

Anca Mustata
University College Cork

Families of manifolds with large symmetry groups

In this talk we discuss families of complex projective varieties with relatively large groups of symmetry, which can be found as moduli spaces of objects in highly symmetric complex projective hypersurfaces. We discuss special families of (n-3)-dimensional complex varieties whose automorphism groups lie inside the (n+1)-th symmetric group. A particular case is the Wiman-Edge pencil of genus 6 complex projective curves. First found in a paper in 1895 by A. Wiman, its modular interpretation was first found by Ph. Candelas, X. de la Ossa, B. van Geemen, D. van Straten in 2012 and explained by Zagier (2014) and I Dolgachev, B Farb, E Looijenga (2018), who proved that every smooth projective curve of genus 6 endowed with a faithful A5-action is equivariantly isomorphic with a member of this pencil.

Richard Smith

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de Leeuw representations of functionals on Lipschitz spaces

Let $\mathrm{Lip}_0(M)$ be the Banach space of Lipschitz functions on a complete metric space (M,d) that vanish at a point $0 \in M$. This has an isometric predual $\mathcal{F}(M) \subset \mathrm{Lip}_0(M)^*$, called the Lipschitz-free (hereafter free) space over M. Free spaces are at the interface between functional analysis, metric geometry and optimal transport theory. They are the canonical way to express metric spaces in functional analytic terms, analogously to how compact Hausdorff spaces can be expressed using $C(K)$-spaces.

We still have a quite poor understanding of the spaces $\mathcal{F}(M)$ and (even more so) their biduals $\mathrm{Lip}_0(M)^*$. Their structure can be probed using the 'de Leeuw transform', which yields representations of each functional on the Lipschitz space $\mathrm{Lip}_0(M)$ in the form of (non-unique) measures on the Stone-Čech compactification $\beta\widetilde{M}$ of $\widetilde{M} := \{(x,y) \in M \times M \,:\, x \neq y\}$.

In this talk we introduce the above and show how topological concepts such as the uniform compactification and 'Lipschitz realcompactification' of (M,d), can be used to study de Leeuw representations of elements of $\mathcal{F}(M)$ and $\mathrm{Lip}_0(M)^*$ and thus shed light on the structure of these spaces. Along the way we introduce a 'metric bidual' of (M,d), whose relationship with (M,d) is analogous to the relationship between a Banach space and its bidual.

This is joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

Filip Strobin
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Rate of convergence in the deterministic version of the chaos game algorithm

The validity of the classical chaos game algorithm for generating images of attractors of contractive iterated function systems can be explained by the fact that, with probability 1, a randomly chosen sequence from a given finite alphabet is disjunctive, meaning that it contains all finite words from that alphabet as its subwords. In particular, given a disjunctive sequence, the generated orbit will approximate the attractor. During my talk I will explain how to measure the rate of convergence of orbits to the attractors and show that additional properties of disjunctive sequences give some control over that rate. On the other hand, I will show that a typical (in the sense of Baire's category and even porosity) disjunctive sequence does not give any control over the rate of convergence. Finally, I will present the result which shows that the situation can be completely different from the probabilistic point of view – in some cases, with probability 1, the rate of convergence of a randomly chosen driver is controlled by the dimension of the invariant measure.

Results related to the deterministic chaos game is joint work with Krzysztof Leśniak and Nina Snigireva, and can be found in

K. Leśniak, N. Snigireva, F. Strobin, it Topological prevalence of variable speed of convergence in the deterministic chaos game, submitted

K. Leśniak, N. Snigireva, F. Strobin, *Rate of convergence in the disjunctive chaos game algorithm*, Chaos 32 (2022), no. 1, Paper No. 013110

Results related to the probabilistic chaos game can be found in

B. Bárány, N. Jurga, I. Kolossváry, *On the convergence rate of the chaos game*, Int. Math. Res. Not. 2023 (2023), no. 5, 4456-4500.

Stephen Watson
York University, Toronto

On the existence of Nash equilibrium

Nash equilibrium is regarded as one of the most important notions in Game Theory. The concept dates back to at least Cournout. However, its current formalization is due to Nash, whose original proof, given in 1950, relies on Kakutani's fixed point theorem. One year later, Nash gave a different proof, which uses Brouwer's fixed point theorem.
The self-contained proof here makes no use of fixed point theorems. Our proof can be split in two parts. The first part introduces two new notions: root function and distributed equilibrium. A root function is a map from the set of mixed strategy profiles to the set of pure strategy profiles. A distributed equilibrium is a subset of mixed strategy profiles that generalizes Nash equilibrium. In the second part, elaborating an argument used by McLennan and Tourky [6], we show that arbitrarily small distributed equilibria always exist. By means of compactness, we obtain the existence of a Nash equilibrium.
Joint work with D. Carpentiere.