Equivalent Values involving Compound Interest

An *equation of value* is usually arranged in order to obtain the required equivalent values. The equation gives the equivalent values of the original obligations and the new obligations on a *comparison date,* sometimes called the *focal date,* at the agreed or present investment market interest rate. The answer for a required equivalent value may vary slightly in simple interest problems, depending on the selection of the comparison date. However, in compound interest problems the selection of the comparison date does not affect the answer.

Example 1

A debt of €200 is due at the end of four years. If money is worth 6% compounded quarterly, what is the value of the debt when it is paid (a) at the end of one year? (b) at the end of six years?

Example 2

A man owes (a) €300 due in three years and (b) €400 due in eight years. He and his creditor have agreed to settle the debts by two equal payments in five and six years, respectively. Find the size of each payment if money is worth 6% compounded semiannually.

Example 3

A man owes (a) €700 due in three years and (b) €1,000 due in eight years. His creditor has agreed for him to pay the debts with a payment of €800 in one year and the remainder in five years. If money is worth 4% compounded annually, what size must the second payment be?

Example 4

A man owes (a) €500 due in two years, and (b) €1,000 with interest at 16% compounded quarterly due in three years. If money is worth 18% compounded semiannually, what single payment seven years hence will be equivalent to the two original obligations?

Equated Date

Example 1

John borrowed some money from Mary as follows: (a) €100 due in one year, (b) €300 due in two years, and (c) €400 due in two and a half years. If money is worth 4% compounded semi-annually, when can John discharge all of his debts by a single payment of €800.

Example 2

Refer to Example 1. If the single payment is €810, when can John discharge all of his debts?

Example 3

Refer to Example 1. If the debt of €400 due in two and a half years is charged at 3% compounded quarterly, when can John discharge his entire debt by a single payment of €800?

Equivalent Rates

If a principal invested at various interest rates will accumulate to the same compound amount in a certain period of time, the rates are said to be equivalent to each other. The various interest rates are thus called equivalent rates. Equivalent rates may be obtained by the use of the effective rate method based on a one-year period.

Example 1

At what nominal rate compounded quarterly will a principal yield interest that is equivalent to 7%?

Example 2

At what nominal rate compounded monthly will a principal accumulate to the same amount as at 8% compounded quarterly?

Example 3

If a principal P, invested at 6% compounded quarterly for three years, will accumulate to the compound amount S, at what nominal rate compounded semiannually will the principal accumulate to the same amount in the same period?

Continuously compounded interest

Although compound interest is usually compounded at regular intervals such as annually, semiannually, quarterly, or monthly, it may be computed more frequently such as every minute, every second, or continuously. Continuous compounding is not commonly used in the actual investment market. However, the concept is theoretically important in analysing financial problems.

Continuously Compounded Interest

To compute interest at a nominal rate compounded continuously,

1. Find the equivalent effective rate
2. Compute the compound interest based on the effective rate.

Example 1

Find the effective rate if money is worth 6% compounded continuously. Here $j=6\%=0.06$.

Example 2

Find the effective rate if money is worth $7\frac{1}{2}\%$ compounded continuously. Here $j=7\frac{1}{2}\%=0.075$.

Example 3

Find the compound amount and the compound interest when €10,000 is invested at 5% compounded continuously for (a) one year and (b) two years.

Applications of Logarithms in Compound Interest Problems

Example 1:

Find the value of S when $S=1000(1+2\%)^{120}$.

Example 2:

Find the value of S when $S=1000(1+2\frac{1}{8}\%)^{10}$

Example 3:

Find the value of S when $S=1000(1+3\%)^{6\frac{1}{2}}$

Example 4:

Find the value of P when $P=2479.27(1+3\%)^{-17}$

Example 5:

Find the value of i when $S=890$, $P=300$ and $n=20$.

Example 6:

Find the value of n when $S=1105$, $P=1000$ and $i=2\%$.