

Some results on products of π -decomposable groups

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Abstract

We present new results regarding products of π -decomposable groups. The starting point of our work is the well-known theorem of Kegel and Wielandt which states the solubility of a group which is the product of nilpotent subgroups. This theorem has been the motivation for a number of results in the literature on factorized groups. In our case we have considered groups $G = AB$ which are factorized as the product of two π -decomposable subgroups A and B , for a set of primes π . A group X is said to be π -decomposable if $X = X_\pi \times X_{\pi'}$ is the direct product of a π -subgroup and a π' -subgroup, where $\pi \cap \pi' = \emptyset$. In this context we have conjectured the following result:

Let π be a set of odd primes. Let the group $G = AB$ be the product of two π -decomposable subgroups A and B . Then $A_\pi B_\pi = B_\pi A_\pi$ and this is a Hall π -subgroup of G .

We show here different approaches to this conjecture; more precisely, when $B = B_\pi$ is a π -group, when A and B have coprime orders and also when A and B are soluble groups, respectively. Moreover we report about the current stage of the research in the general problem. Our results provide extensions of the Kegel-Wieland theorem and, in particular, non-simplicity criteria for finite groups.