

# Global transitions of multistable networks via pinning control: a comparison among control strategies

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February 14, 2019

# Multistable Network

Consider the network

$$\dot{x}_i = -x_i^3 + bx_i + \sigma \sum_j a_{i,j}(x_j - x_i), \quad i = 1, \dots, N. \quad (1)$$

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$$x_i = \sqrt{b} \forall i \quad x_i = -\sqrt{b} \forall i \quad (2)$$

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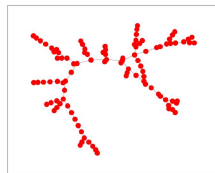
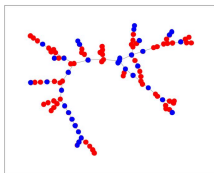
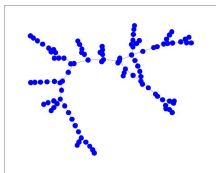
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Controlled system:

$$\dot{x}_i = -x_i^3 + bx_i + \sigma \sum_j a_{i,j}(x_j - x_i) + u_i, \quad i = 1, \dots, N. \quad (3)$$

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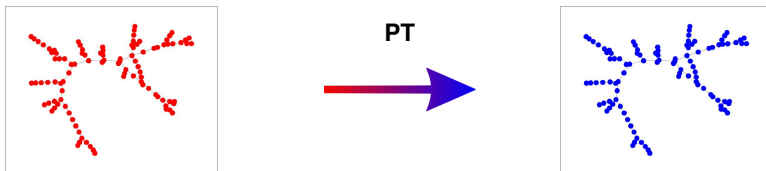
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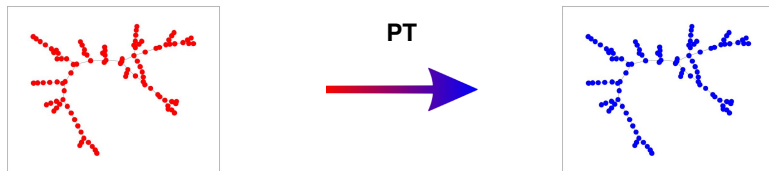


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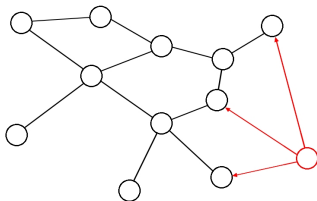
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**Aim:** Compare prerequisites and performances of different control strategies (i.e. rules for determining the control input  $u_i \forall i$ ) that trigger a global transition for the network (3).

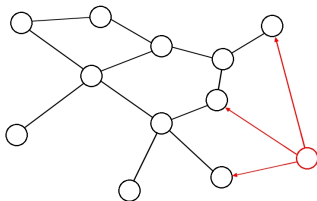
# Pinning control

Control strategies are based on **Pinning Control**, a powerful theory that provides rules for determine controllability of networks.



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Which nodes have to be pinned?



# Control Strategies

## Four control strategies

- *Random*

- 1) A random uncontrolled node is pinned. Go to 2.
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- *Furthest from Synchronization*

- 1) The first pinned node is selected randomly. Go to 2.
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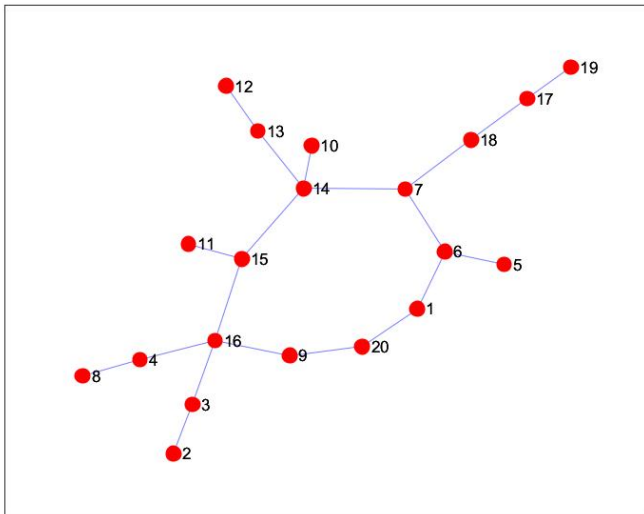
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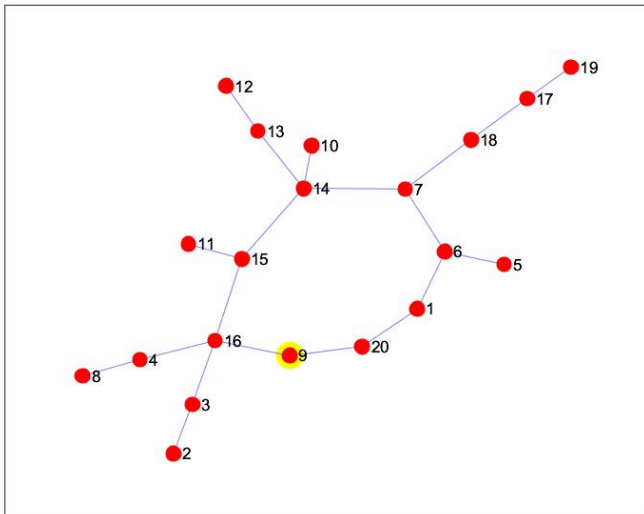
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- *Tree-Spreading Algorithm*

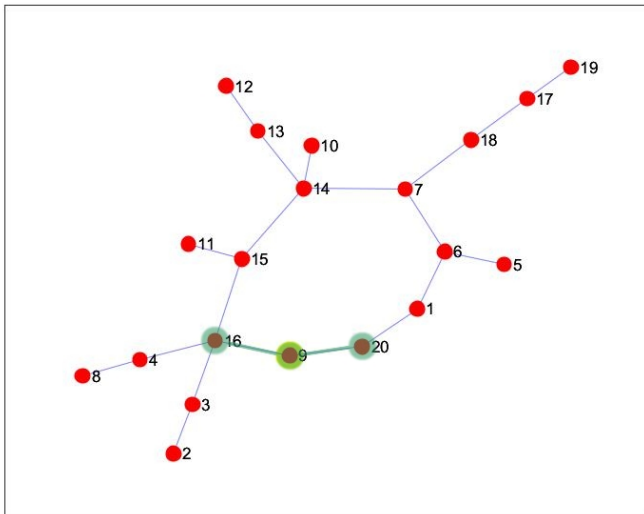
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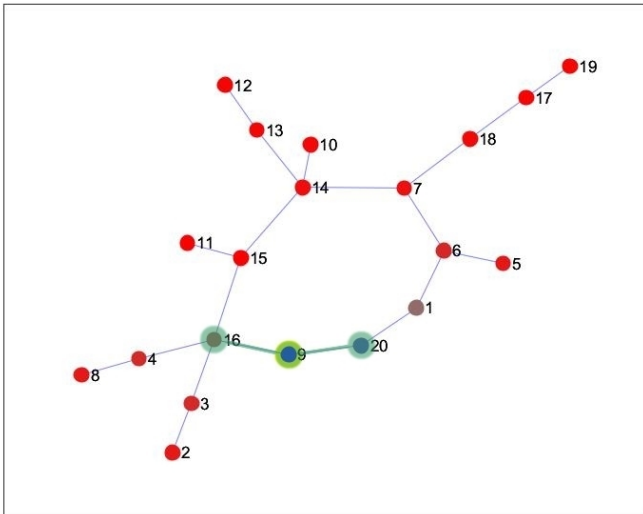
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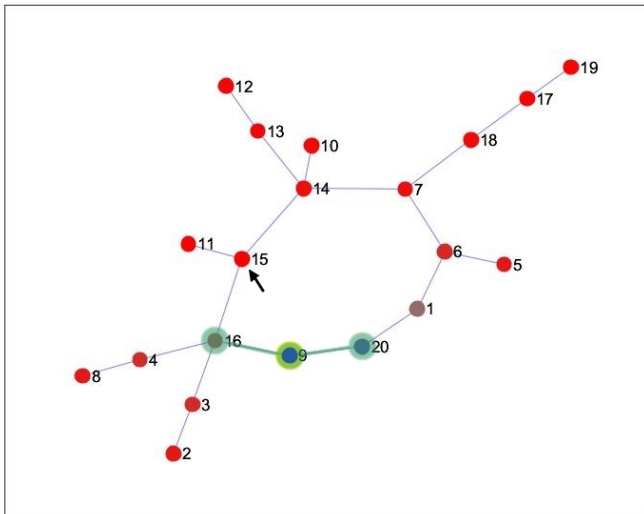


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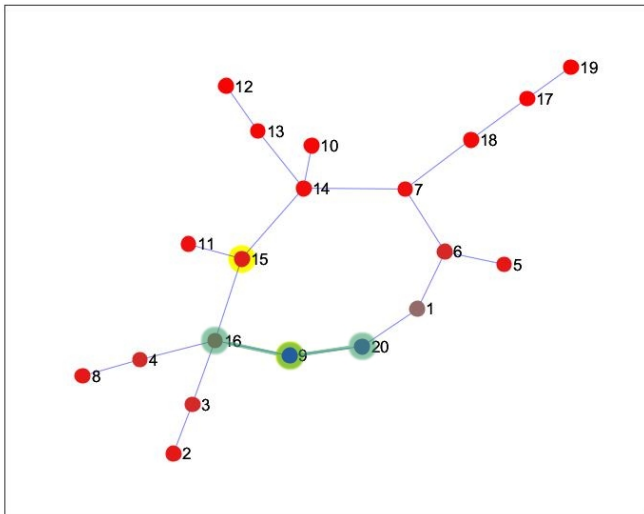




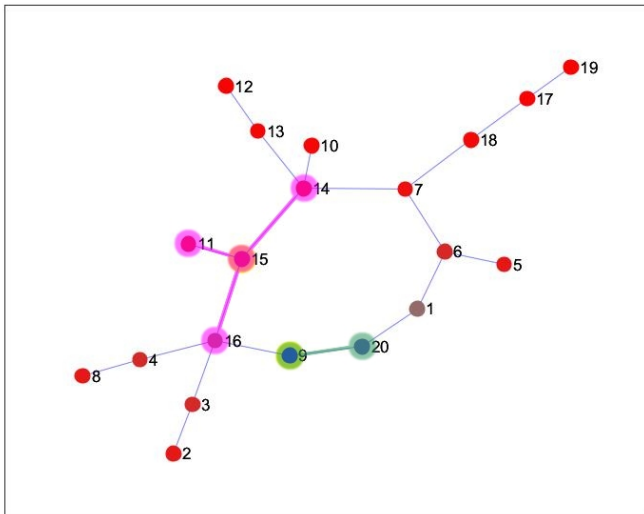
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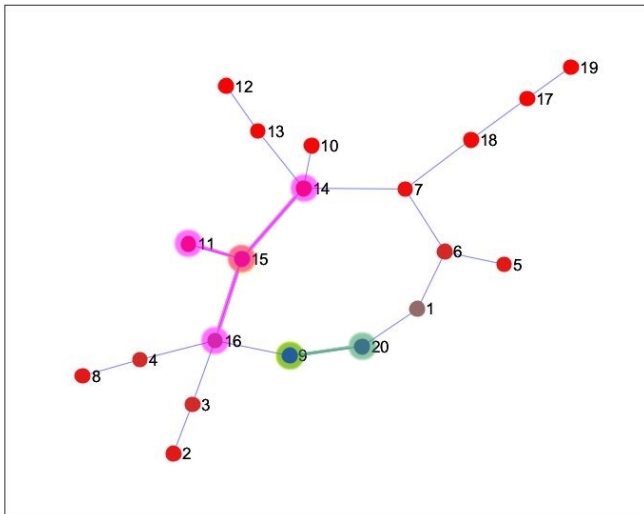
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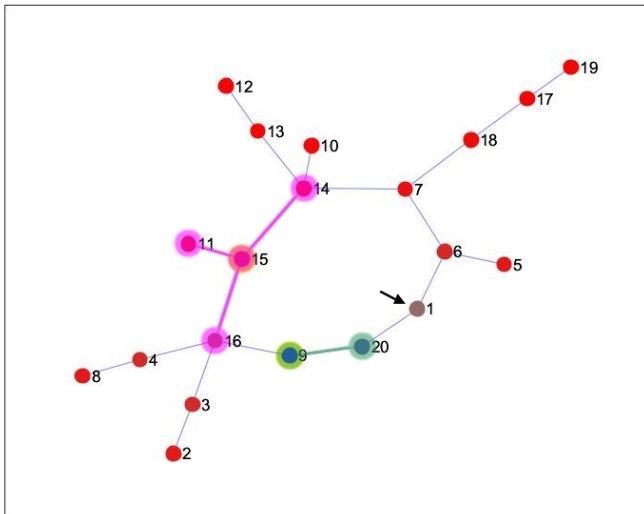
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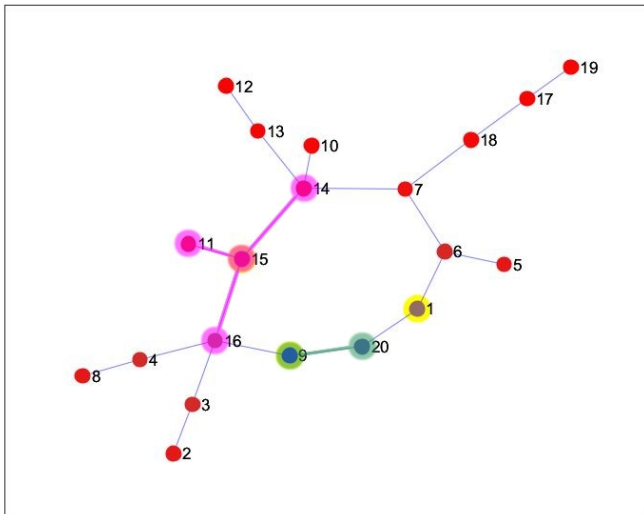
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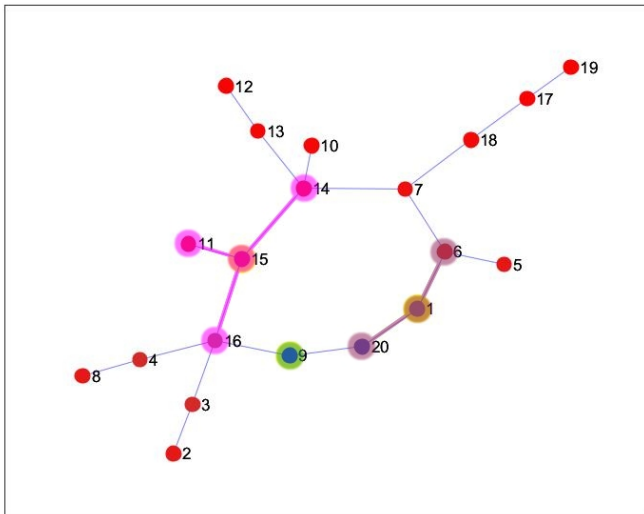
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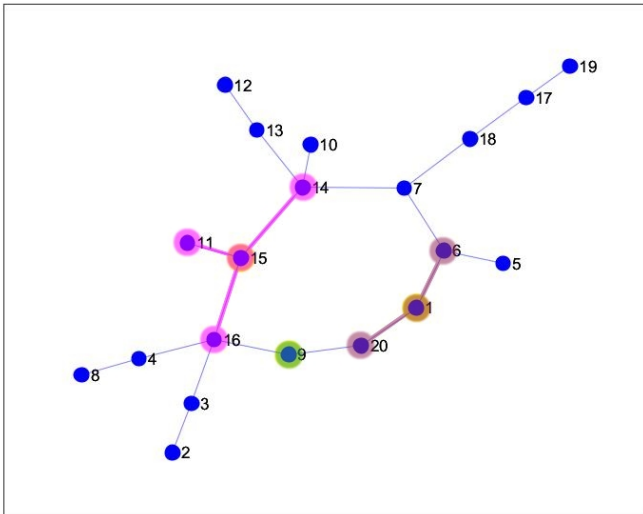
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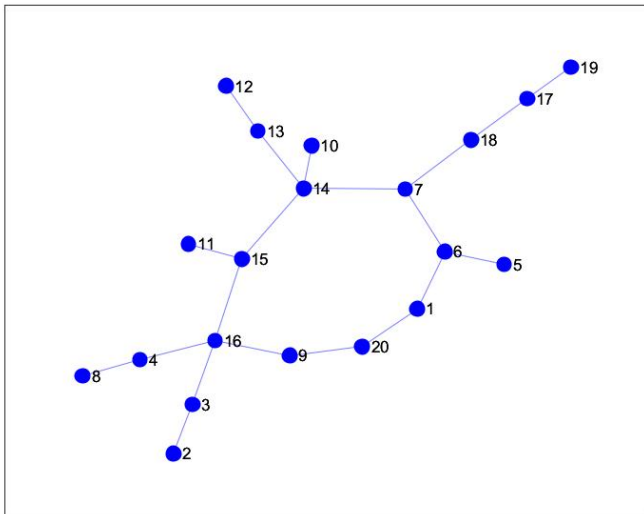


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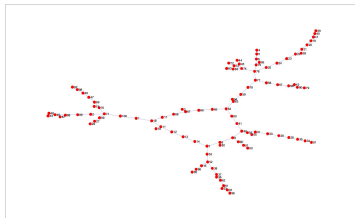
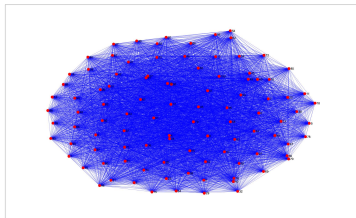
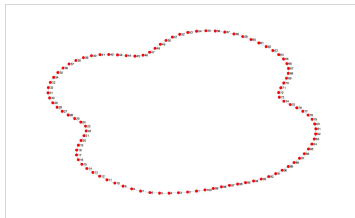
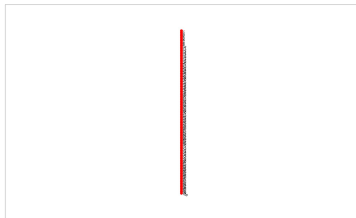


# Tree-spreading control algorithm



# Topologies

We compare 4 topologies with 100 nodes:



# Watts-Strogatz Network - Random

# Watts-Strogatz Network - Highest

# Watts-Strogatz Network - Furthest

# Watts-Strogatz Network - Tree-spreading

# Results

<i>STRATEGY</i>	<i>Centralized</i>	<i>Topology based</i>
<i>Random</i>	Yes	No
<i>Highest</i>	Yes	Yes
<i>Furthest</i>	Yes	No
<i>TreeSpreading</i>	No	No

	Line		Ring		Complete		WattsStrogatz	
	t	pn	t	pn	t	pn	t	pn
<i>Random</i>	2756	38	3238	26	20	2	1640	61
<i>Highest</i>	2893	25	2731	21	20	2	1334	20
<i>Furthest</i>	626	11	2827	12	20	2	1632	17
<i>TreeSpreading</i>	8979	47	4858	46	20	2	1176	45

pn: pinned nodes

# The End