Wrinkling instabilities in soft dielectric plates

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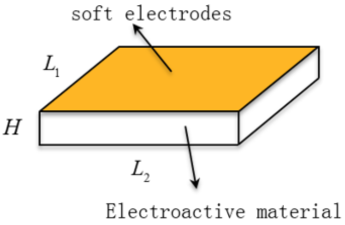
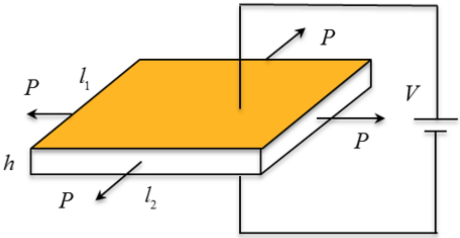
Soft Dielectrics

Soft dielectric materials are smart materials that deform elastically in the presence of an electric field.





They are modelled by coupling the equations of electrostatics with those of non-linear elasticity.

Soft Dielectrics

These materials can be used to produce actuators, artificial muscles or wearable electronics.

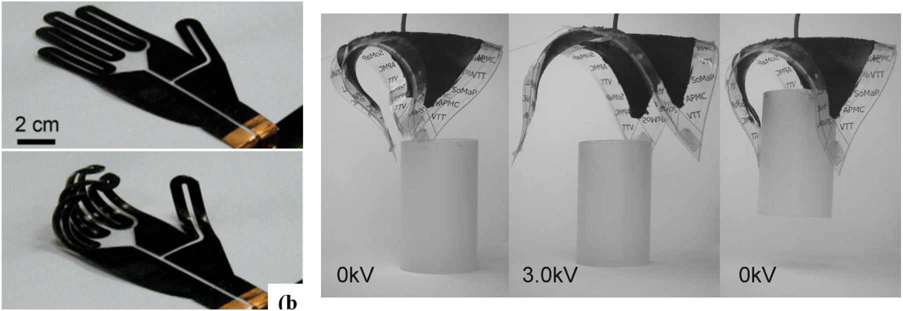


Figure: Applications of dielectric elastomers (Li et al. 2015; Kofod et al. 2007)

Snap-through

Large deformations are achieved using the **snap-through** instability.

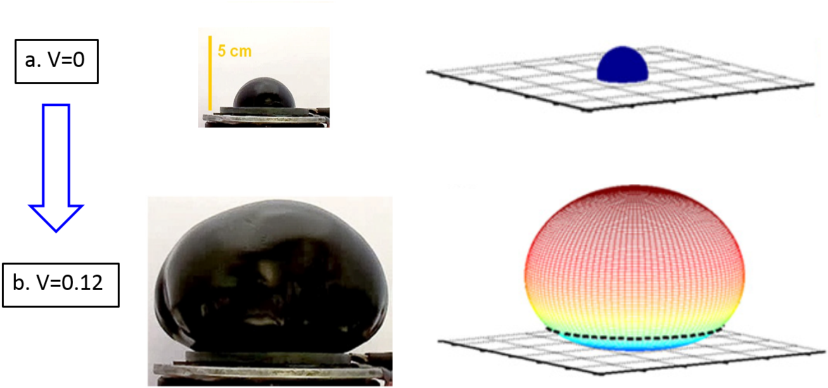
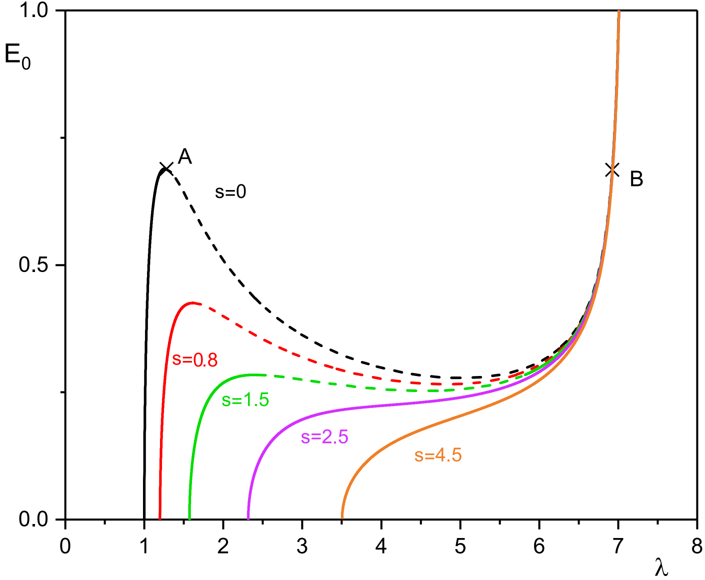
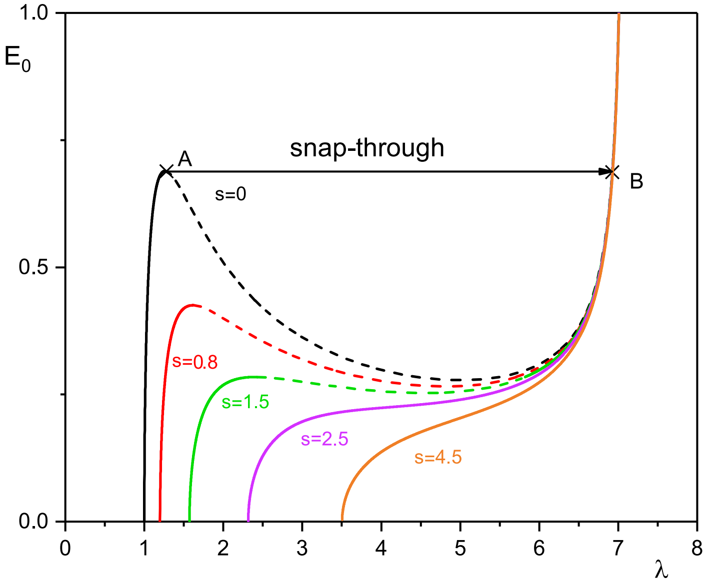


Figure: Experimental evidence of snap-through instability, with area expansion of 1692% (Li et al. 2013)

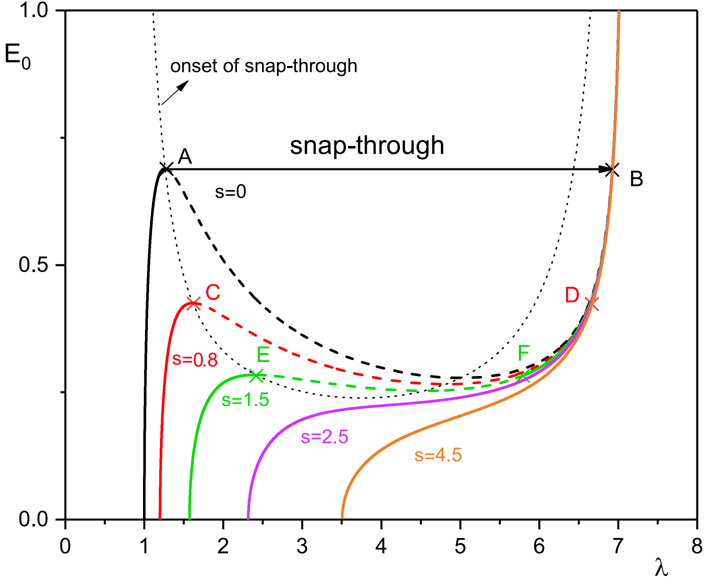
Snap-through



Snap-through



Snap-through



Wrinkling

Snap-through is difficult to achieve in practice, as the material first breaks down or **wrinkles** form.

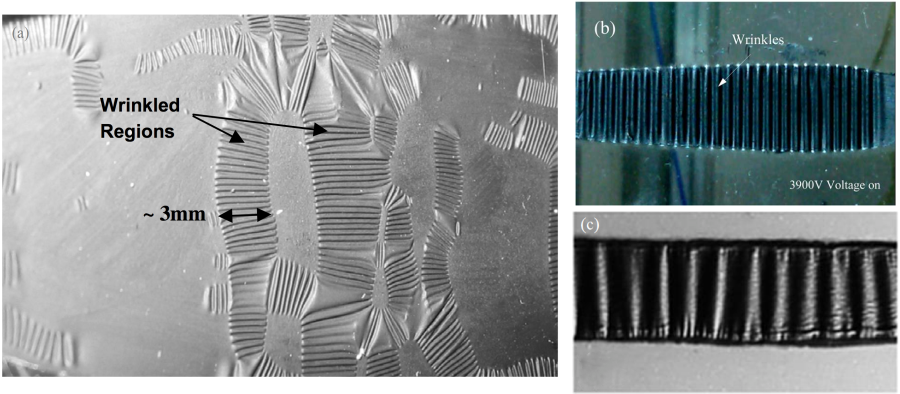
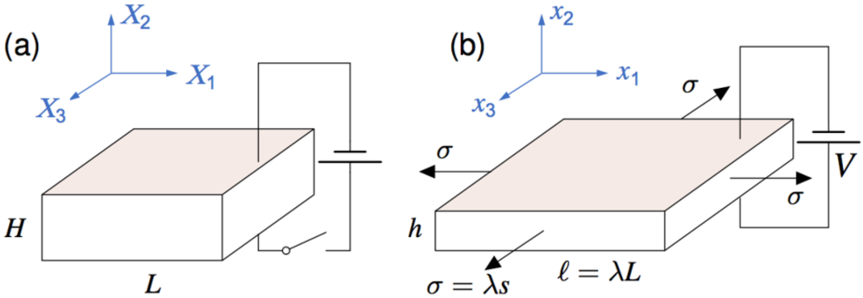


Figure: Experimental evidence of electro-mechanical wrinkling instability (Plante and Dubowsky 2006; Liu et al. 2016; Pelrine et al. 2000)

Setup of Model

Consider a rectangular plate of soft dielectric material that is **stretched equally** along its lateral directions, principal stretches $\lambda_1 = \lambda_3 = \lambda$, $\lambda_2 = \lambda^{-2}$.



We apply a voltage across the thickness direction so that the electric field $\boldsymbol{E}_L = (0, E_0, 0)$.

Setup of Model

We focus on the **Gent dielectric**, which has energy density

$$\Omega = -\frac{J_m}{2} \ln\left(1 - \frac{(2\lambda^2 + \lambda^{-4} - 3)}{J_m}\right) - \frac{1}{2}\lambda^4 E_0^2,$$

where J_m is a stiffening parameter.

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where J_m is a stiffening parameter.

The **nominal stress** is given by

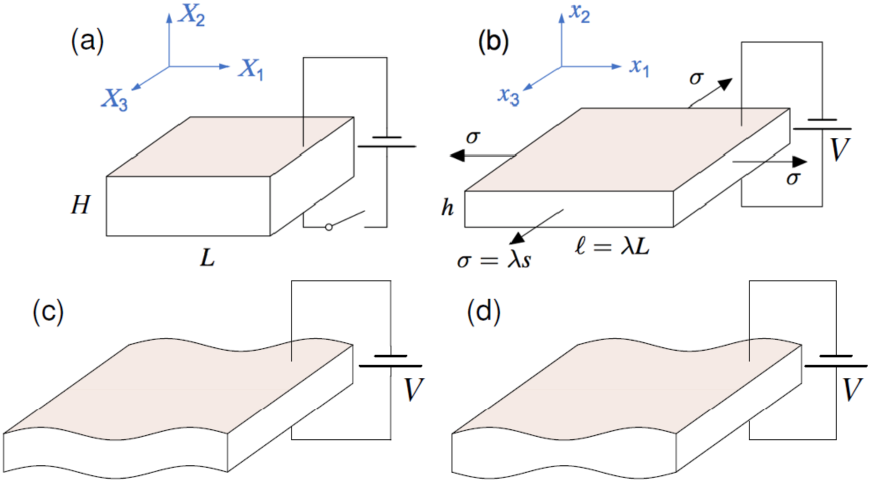
$$s = \frac{1}{2}\frac{\partial \Omega}{\partial \lambda},$$

so that the relationship between voltage and stretch is

$$E_0^2 = \frac{\lambda^{-2} - \lambda^{-8}}{1 - (2\lambda^2 + \lambda^{-4} - 3)/J_m} - \lambda^{-3} s.$$

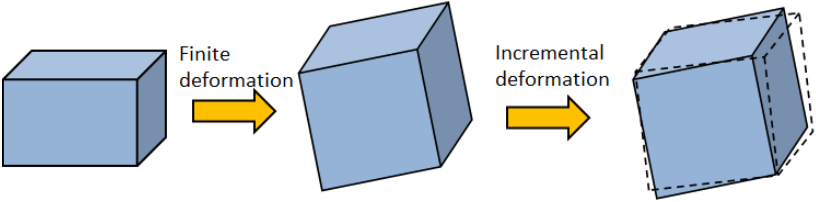
Wrinkling Modes

The plate can wrinkle into **antisymmetric** or **symmetric** modes.



Incremental Deformations

We superpose a small **incremental deformation** denoted $\boldsymbol{u}$ onto a finite deformation.



If the incremental motion satisfies the **incremental equilibrium equations and boundary conditions**, then the material wrinkles.

Incremental Deformations

We linearise the equations and solve the problem using the **Stroh formulation**, which reduces the problem to solving the following equation,

$$\boldsymbol{\eta}' = i\boldsymbol{N}\boldsymbol{\eta},$$

where prime denotes differentiation w.r.t. x_2, the thickness direction.

This results in an eigen-problem for the eigenvalues and eigenvectors $\boldsymbol{\eta}$ of the Stroh matrix $\boldsymbol{N}$.

Results for Gent dielectric

For **thin-plates** ($h \to 0$), wrinkling occurs when

$$E_0^2 = \frac{\lambda^{-2} - \lambda^{-8}}{1 - (2\lambda^2 + \lambda^{-4} - 3)/J_m}.$$

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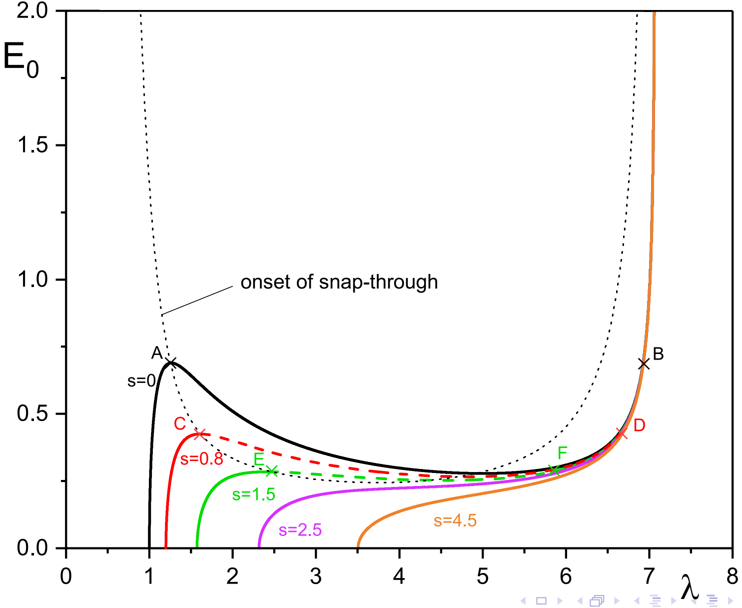
For **thick-plates** ($h \to \infty$), wrinkling occurs when

$$2\lambda(\lambda^9 + \lambda^6 + 3\lambda^3 - 1)W' + 4(\lambda^6 - 1)^2 W'' = \lambda^9(\lambda^3 + 1)E_0^2 \sqrt{1 + 2(\lambda - \lambda^{-2})^2 \frac{W''}{W'}},$$

where

$$W' = \frac{1}{2\left[1 - (2\lambda^2 + \lambda^{-4} - 3)/J_m\right]}, \qquad W'' = \frac{1}{J_m} W'^2.$$

Results for Gent dielectric



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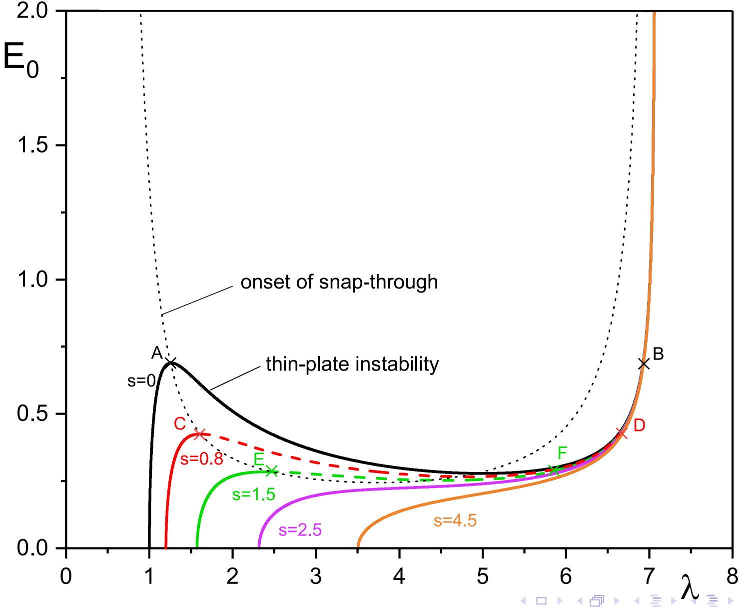

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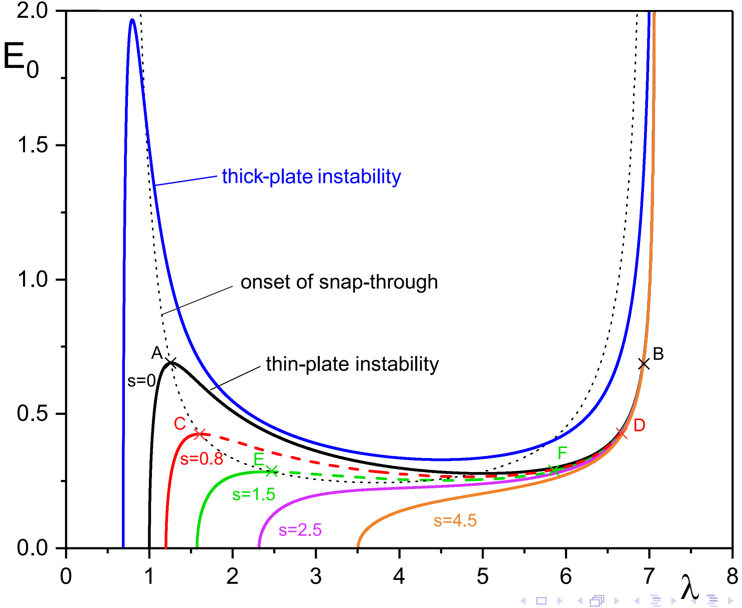

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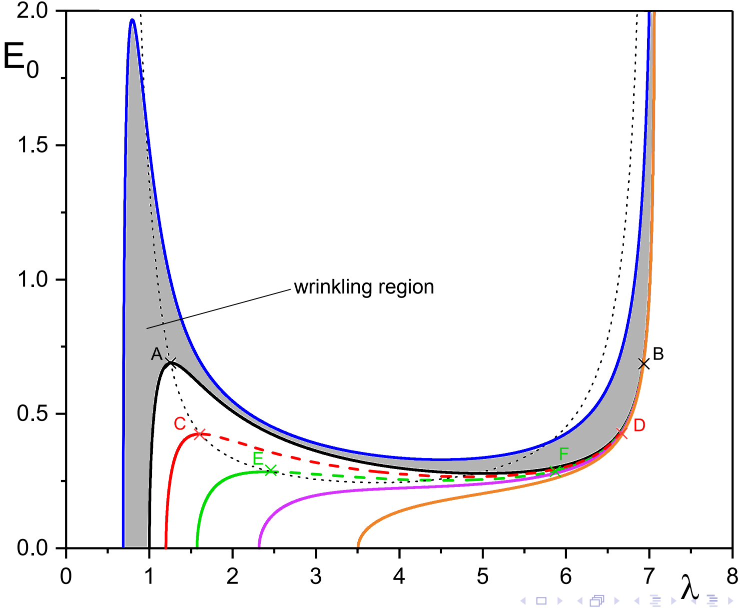

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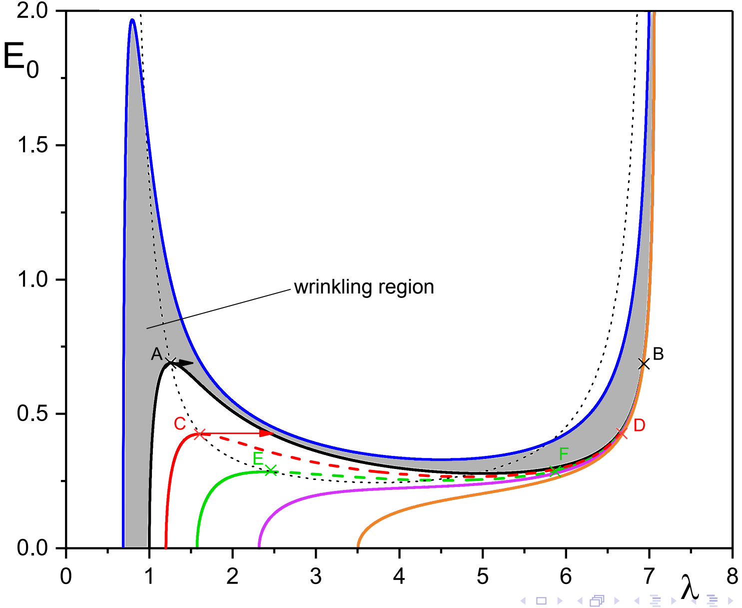

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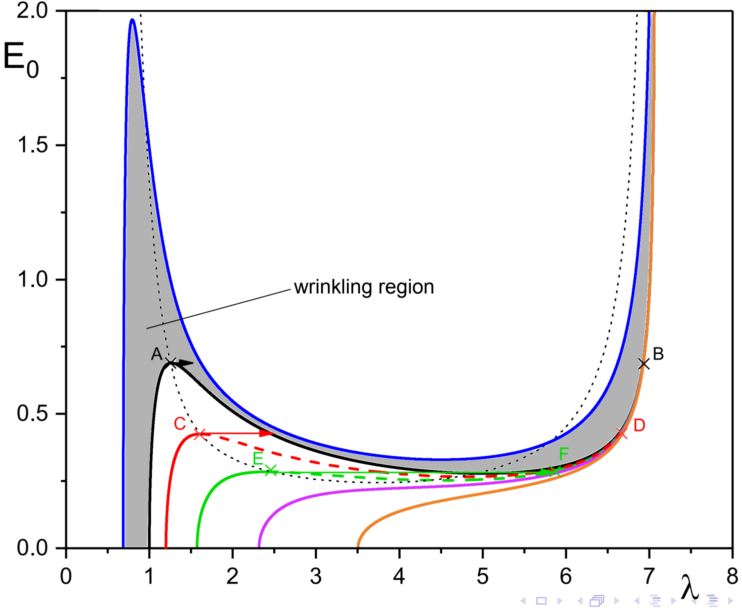
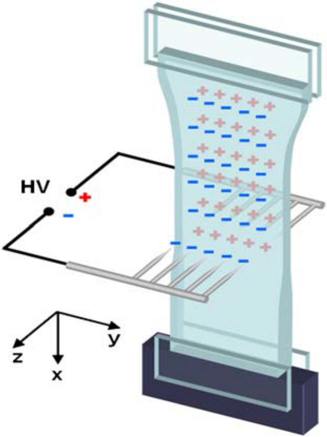


Charge-controlled actuation

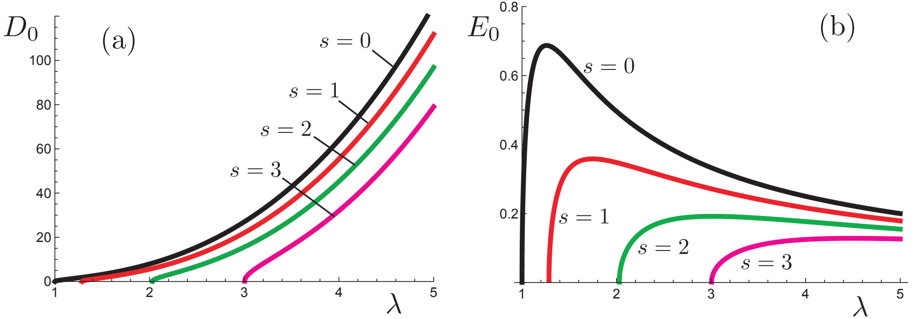


(Keplinger et al. 2009)

- Electric field induced by **spraying charges** of opposite signs to the lateral faces
- **Can wrinkles appear in charge-controlled plates?**

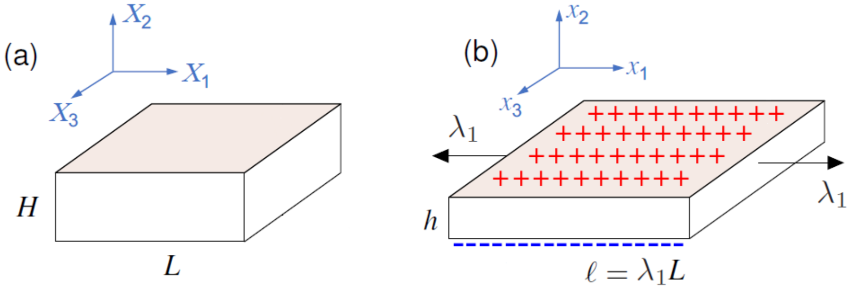
Charge-controlled actuation

Charge-control is **stable with respect to the Hessian** criterion.



Charge-controlled actuation

Consider a rectangular plate of dielectric material, stretched **uniaxially** in the x_1-direction by λ_1, and charges $\pm D_0$ applied on its lateral faces.



The charges **induce an electric field** E_0 in the x_2-direction.

Results for Ideal Dielectric

We focus our attention on **ideal dielectrics**, i.e. materials with energy density

$$\Omega = \frac{1}{2}\left(\lambda_1^2 + \lambda_3^2 + \lambda_1^{-2}\lambda_3^{-2} - 3\right) - \frac{1}{2}\lambda_1^2\lambda_3^2 E_0^2,$$

where λ_3 is the stretch in the x_3-direction.

We can then find the following expression for the **charge** D_0

$$D_0 = -\frac{\partial\Omega}{\partial E_0}.$$

Results for Ideal Dielectric

For **thin-plates** ($h \to 0$), wrinkling occurs when

$$D_0^2 = \lambda_1^4 \lambda_3^2 - 1.$$

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For **thick-plates** ($h \to \infty$), wrinkling occurs when

$$D_0^4 - \left(\lambda_1^4 \lambda_3^2 + 3\lambda_1^2 \lambda_3 - 2\right) D_0^2 - \left(\lambda_1^6 \lambda_3^3 + \lambda_1^4 \lambda_3^2 + 3\lambda_1^2 \lambda_3 - 1\right) = 0.$$

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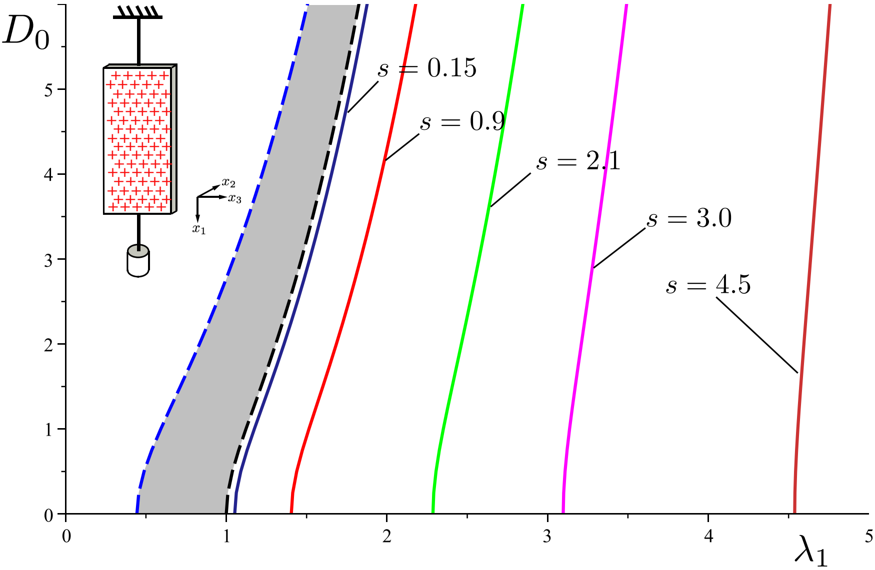
We solve these conditions simultaneously with the loading curve,

$$D_0^2 = \lambda_1^2 \lambda_3^4 - 1,$$

in order to plot D_0 in terms of λ_1.

Results for Ideal Dielectric



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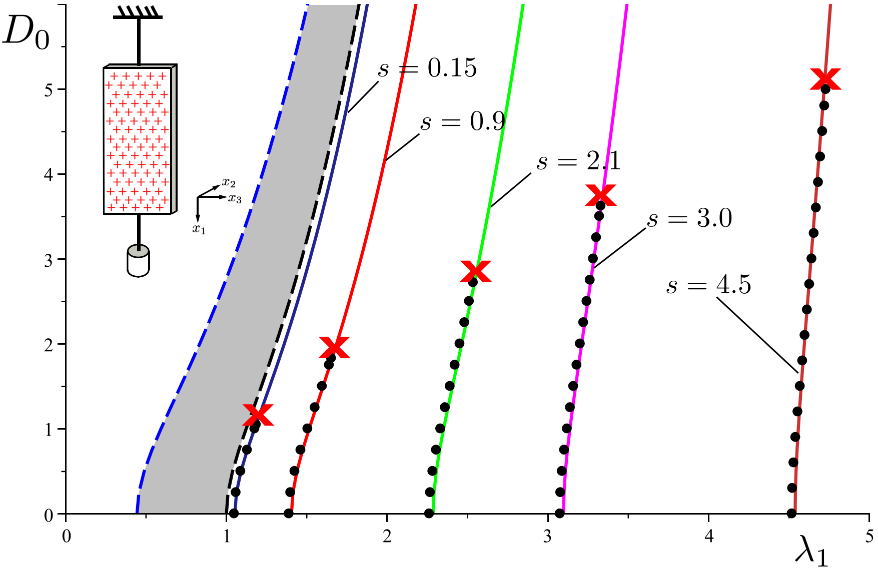
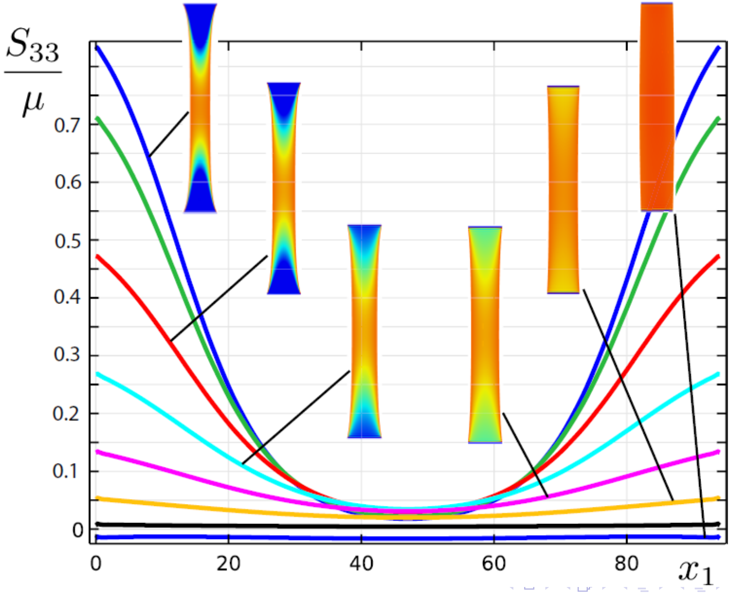


Why do the FE Simulations fail?



Conclusion

- For a real plate, the critical stretch is confined between the thin-plate and thick-plate limits.
- For voltage-controlled actuation, the plates can **wrinkle in both contraction and extension**.
- Charge-control is **geometrically stable**.
- Overall, charge-control is **more stable than voltage-control**.

Y. Su, H. Conroy Broderick, W. Chen, M. Destrade, JMPS, 2018
H. Conroy Broderick, M. Righi, M. Destrade, R.W. Ogden, *preprint* 2019