Charge-controlled dielectric plates

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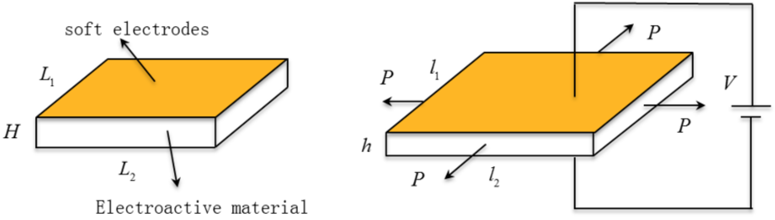
Soft Dielectrics

Soft dielectric materials are smart materials that deform elastically in the presence of an electric field.



They are modelled by coupling the equations of electrostatics with those of non-linear elasticity.

Soft Dielectrics

These materials can be used to produce actuators, artificial muscles or wearable electronics.

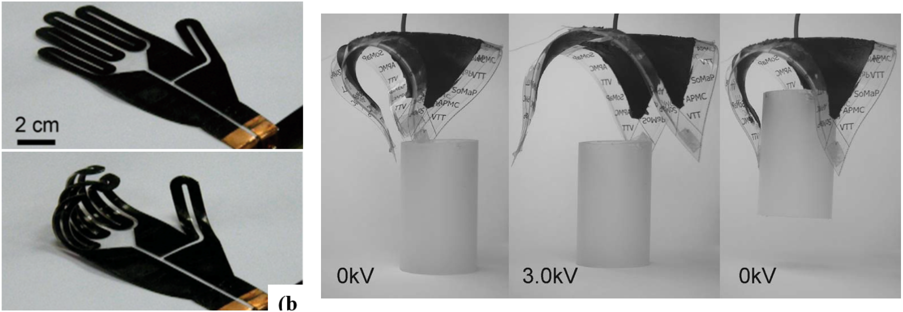


Figure: Applications of dielectric elastomers (Li et al. 2015; Kofod et al. 2007)

Charge-controlled actuation

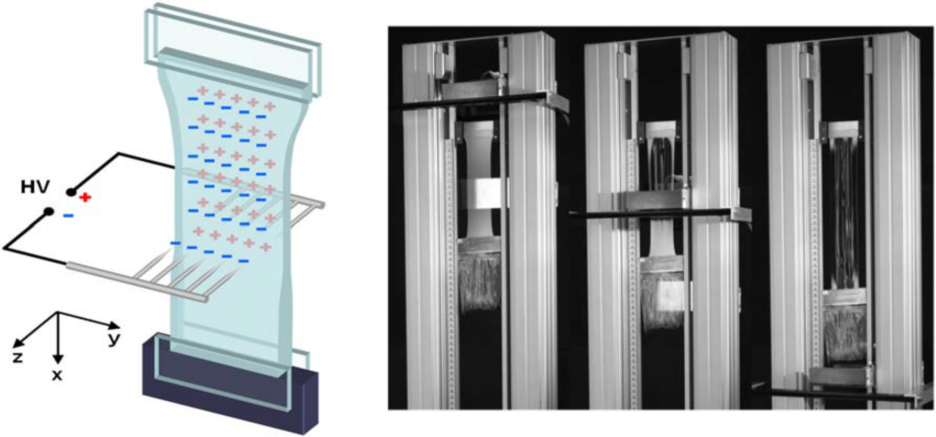


Figure: Charge-control actuator (Keplinger et al. 2009)

Charge-controlled actuation

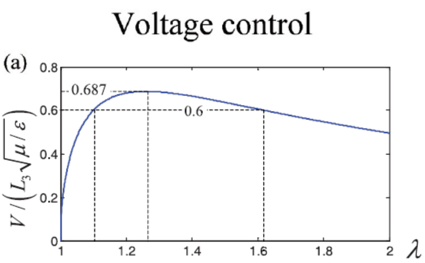


Figure: Comparison of voltage and charge control (Li et al. 2011)

Wrinkling

But are charge-controlled plates geometrically stable?

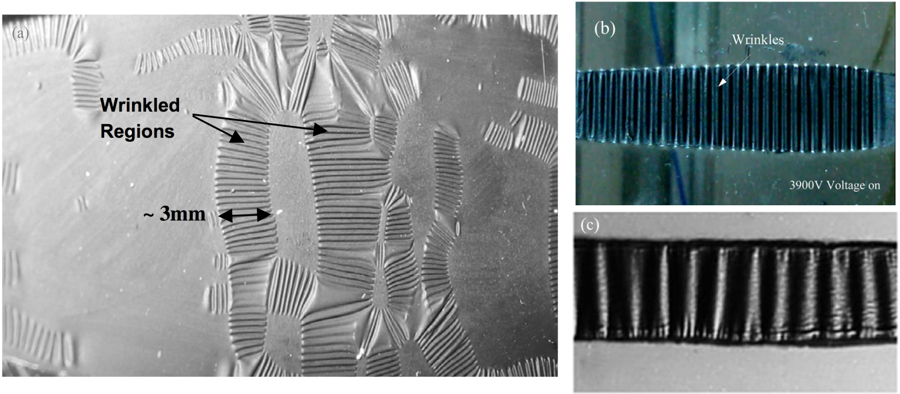
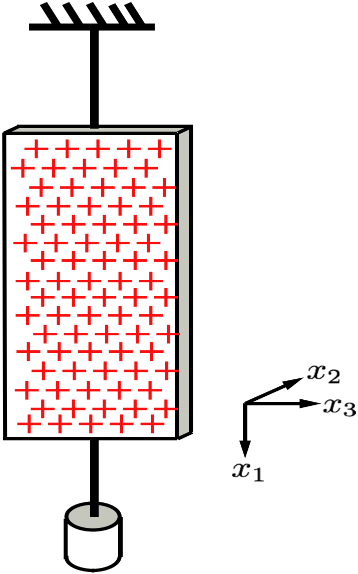


Figure: Experimental evidence of electro-mechanical wrinkling (Plante and Dubowsky 2006; Liu et al. 2016; Pelrine et al. 2000)

Setup of Model

Consider a rectangular plate of soft dielectric material **pre-stretched by a dead weight** so that the stretch is λ_1.

We apply charges $\pm D_0$ to opposite faces of the plate, which induces an electric field E_0 across the thickness.



Setup of Model

We look for solutions in the neighbourhood of the deformation.

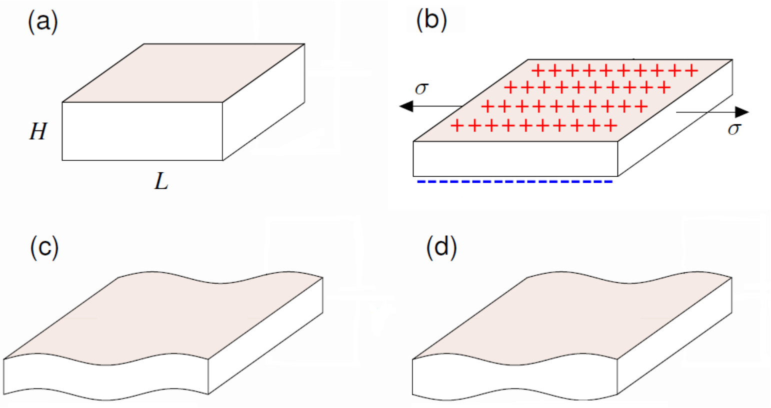


Figure: Antisymmetric and symmetric wrinkles in a soft dielectric plate

Setup of Model

We linearise the equations and solve the problem using the **Stroh formulation**, which reduces the problem to solving the following equation,

$$\boldsymbol{\eta}' = i\boldsymbol{N}\boldsymbol{\eta},$$

where prime denotes differentiation w.r.t. x_2, the thickness direction.

This results in an eigen-problem for the eigenvalues and eigenvectors $\boldsymbol{\eta}$ of the Stroh matrix $\boldsymbol{N}$.

Results for the Ideal Dielectric

We specialise to the **ideal dielectric**, so that the energy density is

$$\Omega = \frac{1}{2}\left(\lambda_1^2 + \lambda_3^2 + \lambda_1^{-2}\lambda_3^{-2} - 3\right) - \frac{1}{2}\lambda_1^2\lambda_3^2 E_0^2,$$

where λ_3 is the stretch in the x_3 direction.

We use the connection $E_0 = \lambda_1^{-2}\lambda_3^{-2} D_0$ to present our results in terms of the charge applied.

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$$D_0^2 = \lambda_1^4 \lambda_3^2 - 1.$$

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We solve these conditions simultaneously with the loading curve,

$$D_0^2 = \lambda_1^2 \lambda_3^4 - 1,$$

in order to plot D_0 in terms of λ_1.

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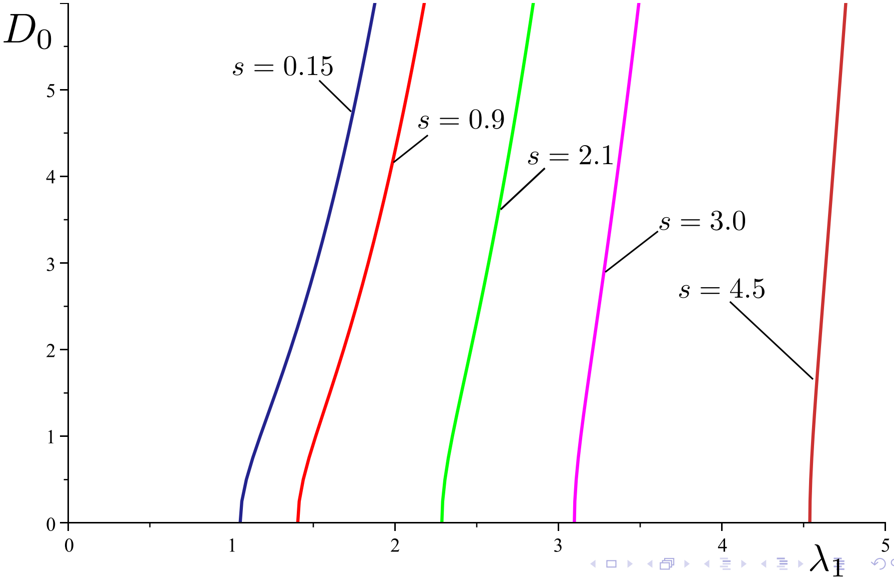

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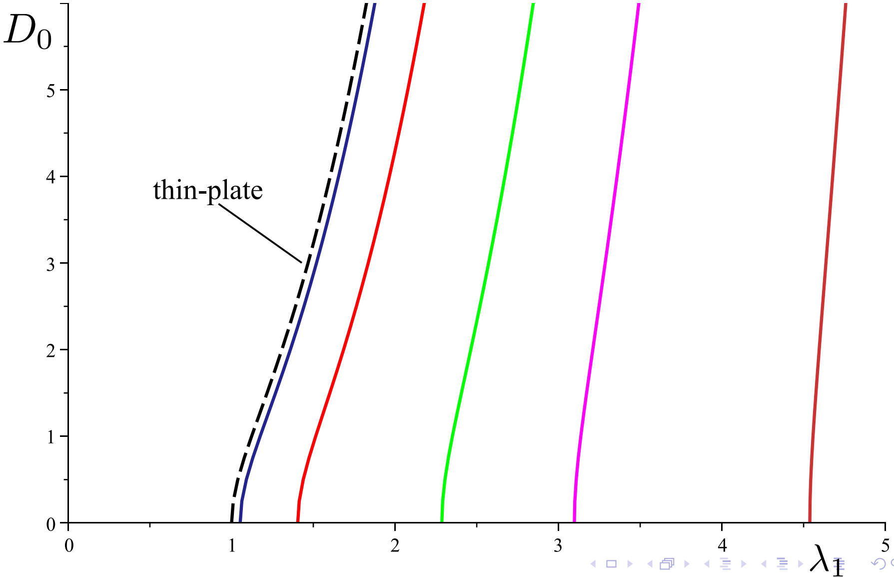

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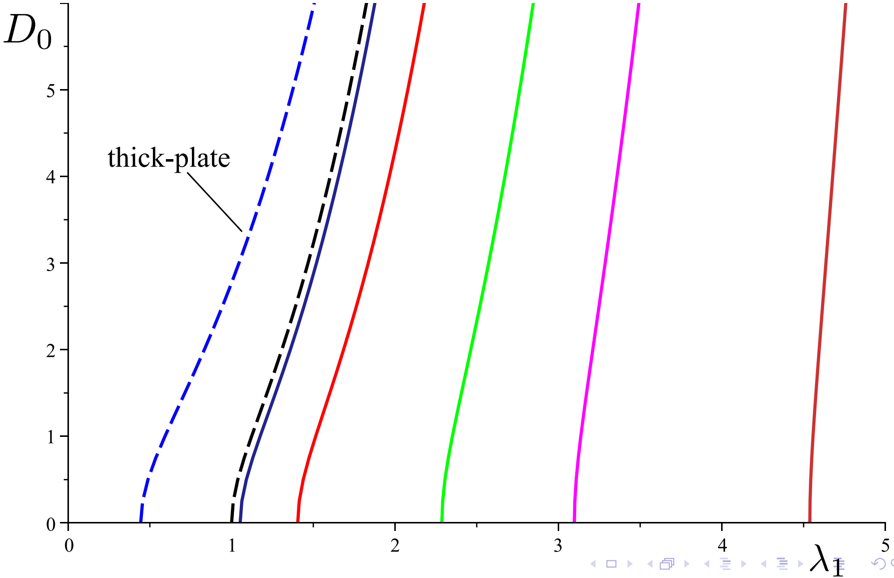

Results for the Ideal Dielectric