

## JOHN TODD AND THE DEVELOPMENT OF MODERN NUMERICAL ANALYSIS

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ABSTRACT. The purpose of this article is to mark the centenary of the birth of John Todd, a pioneer in the fields of numerical analysis and computational science. A brief account is given of his early life and career, and that of his wife, Olga Taussky, including experiences during World War II that led to him engaging with the then developing field of numerical analysis. Some of his contributions to the field, and the contexts in which they arose, are described.

### 1. BEFORE THE WAR

John (Jack) Todd's long and eventful life began on May 17th, 1911 in Carnacally, County Down. I give only an outline of these events here, and refer the interested reader to [1, 2, 5] for further details.

Having attended Methodist College in Belfast, Todd studied at Queen's University Belfast from 1928 to 1931, where A.C. Dixon was professor of Mathematics. He then went to Cambridge, but could not enrol for a bachelor's degree since he had not studied Latin, and so became a research student instead. He was supervised by J.E. Littlewood, who disapproved of the notion of doctoral degrees, so Todd never completed one. He worked under Littlewood's guidance on transfinite superpositions of absolutely continuous functions [25, 26].

He returned to lecture in Queen's University Belfast in 1933, working with J.G. Semple who had recently been appointed as professor. When Semple moved to King's College London in 1937, he invited

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Todd to join him. Initially Todd taught aspects of real analysis there, particularly measure theory, but when another professor became ill while teaching a course in group theory, he was asked to take over.

He developed an interest in the area — enough to attend seminars on the topic at other London Colleges, and tackle a challenging research problem. That in turn led him to contact Olga Taussky who was at Westfield College, leading to a personal and professional partnership that was to last for nearly 60 years.

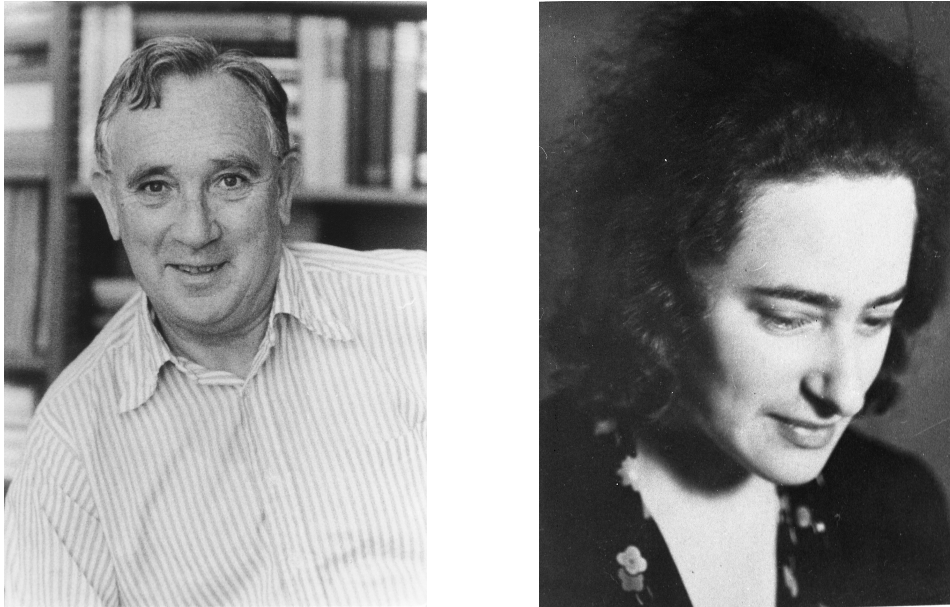


FIGURE 1. John Todd, 1977 and Olga Taussky, (circa 1932). (Source: Archives of the Mathematisches Forschungsinstitut Oberwolfach)

## 2. TAUSKY AND TODD

Olga Taussky was a highly prolific and influential mathematician: she authored roughly 200 research articles and supervised the research of 14 graduate students; she was a founding editor of the journals *Linear Algebra and its Applications* and *Linear and Multilinear Algebra*; she received many awards and distinctions, including election as vice-president of the American Mathematical Society in 1985.

Taussky was born in 1906 in Olmütz, in the Austro-Hungarian empire. She studied in Vienna, initially majoring in both mathematics and chemistry, the latter being related to the family business.

However, she soon focused her attention on mathematics, eventually completing a doctorate in Vienna, under the supervision of Philip Furtwängler, a noted number theorist who contributed significantly to the development of class field theory.

In 1931 she moved to Göttingen, primarily to work with Richard Courant on editing Hilbert's papers on number theory, while also assisting Courant and Emmy Noether with their courses. However, the rise of antisemitism resulted in many academics in German universities, including Courant, Noether and Taussky, being forced to leave their positions. After a short time in Cambridge, Courant became a professor at New York University in 1936, while Noether moved to Bryn Mawr College in Pennsylvania. Taussky was awarded a three year research fellowship from Girton College in 1935, and decided to spend the first year of that in Bryn Mawr with Noether.

After applying for numerous positions, Olga Taussky was eventually appointed to a teaching post at Westfield College, a constituent of the University of London. In 1937 she met Todd and within a year they married, somewhat inauspiciously, on the day the Munich Agreement was signed. Their first joint articles, which show Taussky's emerging interest in the developing field of matrix theory (e.g., [21, 22]) were written in a bomb shelter in London. Their final joint paper [23], a historical note on links between the celebrated method of Cholesky and work of Otto Toeplitz, was completed shortly before Taussky's death in 1995.<sup>1</sup>

For more details of Taussky's life and career, see [17] and [47], and the autobiographical articles [24] and [20]: the former is primarily concerned with her life and experiences, the latter with her contributions to matrix theory. Her contributions to other areas of algebra are discussed in [16].

### 3. DURING THE WAR

With the outbreak of World War II, and their colleges evacuated from London, Taussky and Todd moved to Belfast where they both

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<sup>1</sup>The manuscript was originally submitted in 1995. Following the death of Taussky later that year, the manuscript was "lost" for several years. Some years after, there was renewed interest in the origins of Cholesky's method, including the discovery by Claude Brezinski of an original, unpublished, hand-written note by Cholesky describing it. So in 2005, Todd resubmitted the paper for publication in *Numerical Algorithms*.

taught for a year. Eventually they were to return to London and to work in scientific war jobs.

Taussky worked on aerodynamics at the National Physics Laboratory with the Ministry of Aircraft Production. Here she learned a great deal about differential equations, which had not interested her much previously, and matrix theory. She worked on problems in flutter [44], expressed as hyperbolic differential equations, and developed a technique that greatly reduced the amount of computational effort required to estimate the eigenvalues of certain matrices. Her idea was to use the simple, but very powerful, idea introduced by Geršgorin [12] which shows that the eigenvalues of an  $n \times n$  matrix  $A$  with complex entries are contained in the union of  $n$  disks, where the  $i^{\text{th}}$  disk is centred on  $a_{ii}$  and has radius  $\sum_{j \neq i} |a_{ij}|$ . Taussky then used carefully chosen similarity transforms that reduce the radius of the disks, thus improving the accuracy of the estimate.

Although Geršgorin's work had received some attention, Taussky is often credited with popularising it, for example in [19]. Many generalisations and extensions were to follow, by Taussky and by others; an accessible account of these is given by Varga in [48]. The study of Geršgorin's disks were also a topic of research in the Ph.D. studies of Taussky's Irish student, Fergus Gaines [11].

While Taussky was working on aeronautics, Todd worked with the Admiralty in Portsmouth, initially on ways of counteracting acoustic mines. During that period he was struck by the amount of time that physicists spent doing routine calculations, while mathematicians were attempting to engage with engineering problems:

“This was rather frustrating: physicists were doing elementary computing badly and mathematicians like me were trying to do physics. I thought that I could see a way to improve this mismatching” [1].

Todd persuaded his superiors to reassign him to London to establish what became the Admiralty Computing Service, centralising much of numerical computations for the naval service. He remained there until 1946.

In 1942 John von Neumann visited the Admiralty to inspect some of their ballistic facilities in connection with his work on developing the atomic bomb at Los Alamos. Todd was given the task of accompanying him, and introduced him to their computing facility.

This led to the rather remarkable claim, made by von Neumann, that Todd was responsible for getting him interested in computing!

Todd's work with the Admiralty also led to what he considered his greatest contribution to mathematics. In 1945 he was part of a small group that visited Germany to investigate mathematics that might be of interest to the Navy, such as the work of Konrad Zuse on programmable computers. The group also visited Oberwolfach, which they had heard was being used as a mathematics research centre. They arrived just in time to prevent it from being looted by Moroccan soldiers. Because it was in the French zone of occupation, Todd subsequently travelled to Paris to persuade the French government to maintain it. It later developed into a world renowned centre for mathematical research. For a lively recounting of the adventures of this time, see [41].

#### 4. CONVERSION TO NUMERICAL MATHEMATICS

Even before setting up the Admiralty Computing Service, Todd had developed an interest in the topic of computing, initially prompted by Alan Turing's work on computable numbers [45], and through contacts with the British Association for the Advancement of Science (BAAS), which was mainly concerned with making tables—often regarded as the primary goal of early scientific computing.

When he returned to Kings College in 1946, he taught the first course there on numerical mathematics. There were no text books for this developing area, so Todd developed his own notes. This included a section on the solution of systems of linear equations, featuring the method of Cholesky, which at the time was not well known in the mathematics literature.<sup>2</sup> It was through this course that Leslie Fox and colleagues at the Mathematics Division of the National Physics Laboratory became aware of this now standard method.

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<sup>2</sup>André-Louis Cholesky was a French military officer. He developed his eponymous method for solving linear systems involving Hermitian, positive-definite matrices when he was engaged in computing solutions to certain least-squares problems that arise in geodesy. Compared to other approaches at the time, it was remarkably efficient—Cholesky reported that he could solve a system of 10 equations, to 5 decimal digits of accuracy, in under five hours! He explained his method to other topographers, but never published it. It was eventually published in a journal on geodesy by a colleague several years after Cholesky's untimely death towards the end of World War I [6]. See also [3].

Around this time, Artur Erdélyi and Todd wrote an article in *Nature* based on their observations of industrial mathematics research [9]. They argued for the foundation of an *Institute for Practical Mathematics* in the U.K. to provide instruction in the “mathematical technology” needed for developments in engineering, mathematical biology and economics. The call was not immediately heeded, but in 1947, Todd and Taussky moved to the United States, at the invitation of John Curtiss, to help establish the new Institute for Numerical Analysis at the National Bureau of Standards. Following an “inspirational” three months visiting von Neumann at Princeton [2], they began working at the INA, located at first at UCLA. They moved to Washington a year later where they stayed for 10 years.

In 1957 Todd and Taussky were offered positions at Caltech: John as Professor of Mathematics, and Olga as a research associate “with the permission but not the obligation to teach” [17].<sup>3</sup> Todd’s appointment was made so as to develop courses in numerical analysis and computation within the mathematics department. They remained at Caltech for the rest of their lives. Taussky died on October 7<sup>th</sup>, 1995. Todd died on May 16<sup>th</sup>, 2007.

## 5. TODD AND NUMERICAL ANALYSIS

Prior to the 1940s, a “computer” was usually understood to be a person who carried out calculations, and the designers of numerical algorithms had in mind the development of methods that were to be implemented by hand. Computing machinery was mainly limited to hand-operated mechanical calculators, such as the 10-digit Marchant Model 10 ACT—the first calculator used in Todd’s course on numerical analysis at King’s College in 1946. The same year, ENIAC (*Electronic Numerical Integrator And Computer*), the first general-purpose, programmable electronic computer, was launched.

From the 1940s, the rapid development of computer hardware was mirrored by rapid developments in the field of numerical analysis: it could be argued that the field emerged as a discipline in its own right between 1940 and 1960. (The term itself is usually credited to

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<sup>3</sup>At the time, Caltech regulations prevented Todd and Taussky from holding professorships concurrently; this was only modified years later as a result of the recognition that Taussky was receiving as one of the country’s leading mathematicians. Taussky was granted tenure in 1963 and a full professorship in 1971.

John Curtiss, and its first use was in the name of the Institute for Numerical Analysis that he founded in 1947). Landmark advances included the development of the Crank-Nicolson method for time-dependent partial differential equations in 1947, and the discovery of the more computationally efficient *alternating direction method* (ADI) by Peaceman and Rachford in 1953. The basis for Finite Element methods, now the most popular approach for numerical solution of partial differential equations in engineering applications, was provided by Courant in 1943, but their full potential was not realised until the 1960s. In 1947, von Neumann and Goldstine produced the first mathematical study of direct numerical solution of linear systems, with particular regard to the effects of round-off error.<sup>4</sup> New algorithms (and their analyses) for the iterative solution of linear systems included successive over-relaxation (SOR) in 1950, and the Conjugate Gradients method in 1952 (though the latter did not achieve significant popularity until much later). The *Fast Fourier Transform* of Cooley and Tukey was developed in 1965.

The American Mathematical Society's journal *Mathematical Tables and Other Aids to Computation* was launched in 1943, and renamed *Mathematics of Computation* in 1960. In 1959, the first journal for numerical analysis, Springer-Verlag's *Numerische Mathematik* was founded, with Robert Sauer, Alwin Walther, Eduard Stiefel, Alston Householder, and John Todd as editors. Todd served on the editorial board for 49 years. The SIAM Journal on Numerical Analysis was founded later, in 1964.

Readers interested in the history of the development of the field of numerical analysis in the 20<sup>th</sup> century should consult the recent article by Grcar [14] which pays special attention of the importance of von Neumann's work, particularly [49], and those who developed its ideas further, including John Todd.

**5.1. Articles.** Todd's first papers in numerical analysis were related to computational linear algebra, and the problems of ill-conditioning of matrices. Suppose  $A$  is a nonsingular matrix and we wish to solve the system  $Ax = b$  by computational means. Simply representing  $b$  with finitely many digits introduces numerical error. In many cases

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<sup>4</sup>Their paper [49] is often cited as the first in modern numerical analysis; others would give that honour to Turing for his 1936 paper on computable numbers [45], while in [38] Todd points to Comrie's 1946 article on the use of general business machines in solving computational problems [7].

of interest, the implementation of standard direct solution algorithm (all variants on Gaussian elimination) on a computer with finite precision greatly magnifies these errors. In [49], von Neumann and Goldstine introduced a quantity they called the “figure of merit” which gives an upper bound on the magnification of the errors. This is usually denoted  $\kappa(A)$  and is now known as the *condition number* of the matrix, a term coined by Alan Turing in a related work written around the same time [46]. If  $\kappa(A)$  is large, then the matrix is said to be *ill-conditioned*. It is usually defined as, for example,

$$\kappa(A) = \|A\| \|A^{-1}\|,$$

where  $\|\cdot\|$  is one’s favourite matrix norm, or as the ratio of the largest to smallest eigenvalue of  $A$ . This latter case can be useful in practice, since it does not require direct knowledge of  $A^{-1}$ . Furthermore, in [49] the “figure of merit” is given as  $\|A\|_2 \|A^{-1}\|_2 = \sigma_n/\sigma_1$  where  $0 < \sigma_1 \leq \dots \leq \sigma_n$  are the singular values of the invertible matrix  $A$ . But since in most of the cases considered,  $A$  is a symmetric positive definite matrix, this is the same as  $\lambda_n/\lambda_1$  where  $0 < \lambda_1 < \dots < \lambda_n$  are the eigenvalues of  $A$ . Turing’s proposed measures included using a scaled Frobenius norm. Todd [27, 29] studied this issue for a matrix arising in the numerical solution of a second-order elliptic problem in two variables by the standard finite difference method, and showed the relationship between several measures of conditioning proposed by Turing, von Neumann and others. His work was instrumental in Goldstine and von Neumann’s quantity becoming accepted as *the* condition number. He went on to study fourth and higher-order problems in [32]. See [14] for a further discussion of this.

Other articles, including [28], are concerned with the stability of finite difference schemes, and propose that such analysis be done based on the matrix analysis of resulting linear systems. Similar ideas, but for several explicit and implicit schemes for time-dependent problems, are found in [34]. A mathematical analysis (explaining experimental results obtained by other authors) for an ADI method is given in [10]. The computation of special functions features in [15], for example, and the efficiency of methods for solving integral equations is considered in [35].

Although numerical analysis is often (and certainly, originally) understood as the mathematical study of computer algorithms for solving mathematical problems, in [33] Todd coined the term “ultramodern numerical analysis” (or “adventures with high speed automatic



digital computing machines”) which, unlike other areas of mathematics of the time, features aspects of experimentation, particularly where rigorous error analysis was not possible for sufficiently complicated problems. A systematic study of such experimentation, using matrix inversion as the main illustration, is given in [18]. An experimental study of a linear solver is given in [30], and of the application of a Monte Carlo method for solving a partial differential equation in [31] (as originally proposed by Courant, Friedrichs and Lewy in their celebrated 1928 paper that includes their famous condition for the stability of explicit schemes for time dependent problems [8]). Given the need to avoid over-extrapolation based on numerical experiments for a limited number of examples, Todd [33] cautioned that “separation of theoretical and applied numerical analysis is undesirable”.

Although many of Todd’s later papers were on the history of computational mathematics, he continued publishing original research into his seventies [4, 42] and eighties [43].

**5.2. Books.** While at the Bureau of Standards, Todd developed a programme to help train mathematicians in the new techniques of computational mathematics. He arranged for experts in the field to give courses in particular topics. At the suggestion of Taussky, the notes from these courses developed into the highly influential *Survey of Numerical Analysis* [36]. The first chapter, a reworking of [33] mentioned above, is titled *Motivation for Working in Numerical Analysis*, and notes that

“the profession of numerical analysis is not yet so desirable that it is taken up by choice; indeed, although it is one of the oldest professions, it is only now becoming respectable”.

He distinguished between what he termed classical, modern and ultramodern numerical analysis. Classical numerical analysis is concerned with solution, by hand, of problems in interpolation, integration, and the approximation of solutions to initial value problems<sup>5</sup>. Modern numerical analysis, on the other hand, is required for the exploitation of automatic digital computation. Finally, ultramodern numerical analysis relies on a combination of rigorous mathematics,

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<sup>5</sup>See the fascinating monograph [13] for details on the fundamental developments of classical numerical analysis.

where problems permit it, and careful study of experimental results for problems whose complexity is beyond existing mathematical theory (for example, one might devise a numerical method for solving a nonlinear differential equation, give a mathematical analysis for a linearised variant and the results of supporting numerical experiments for the full nonlinear problem).

The survey contained contributions from, among others, Morris Newman, Harvey Cohen, Olga Taussky, Philip J. Davis, Werner Rheinbolt and Marshall Hall, Jr.. Topics covered include approximation of functions, the principles of programming, Turing Machines and undecidability, numerical linear algebra, differential equations, integral equations, functional analysis, block designs, number theory, and computational statistics.

Todd did not completely abandon his earliest research interests. His 1963 monograph, *Introduction to the Constructive Theory of Functions* [37] drew from the tradition of classical analysis to present sometimes neglected ideas on Chebyshev theory and orthogonal polynomials, while still providing “some mild propaganda for numerical analysis”.

The courses in numerical analysis that Todd developed at Caltech became the basis for the two volume *Basic Numerical Mathematics*. As educational aspects of the subject developed, most presentations were aimed either at students at graduate or upper undergraduate level, or incorporated computational aspects into introductory courses in linear algebra and calculus. As he explains in Volume 1 [40], Todd aimed to introduce aspects of numerical computation after only the basics of calculus and algebra had been studied. He combined both “controlled numerical experiments”, to reinforce ideas such as convergence and continuity, with “bad examples”, to temper the tendency to rely on numerical experience rather than develop sound mathematical analyses:

“The activities of the numerical analyst are similar to the highway patrol. The numerical analyst tries to prevent computational catastrophes”.

Often, the existence of such “bad examples” is due to the subtle difference between real numbers and those that might be represented

by computer. As an example (see [40, Chap. 3]) consider the divergent harmonic series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots .$$

If this sum is constructed on a computer it will appear to be convergent since, for large enough  $n$ , the computer will not be able to distinguish  $1/n$  from zero.

Volume 1 [40], subtitled “Numerical Analysis”, covers topics in interpolation, quadrature and difference equations, and are complemented by (relatively) elementary programming exercises. Since most programs required are for scalar problems, students were expected to develop a complete implementation of the algorithms themselves.

Volume 2 [39], “Numerical Algebra” deals with direct and iterative methods for solving linear systems of equations, and for the estimation of eigenvalues, with applications to curve fitting and solution of boundary value problems. Because the algorithms require the representation and storage of vectors and matrices, unlike the earlier volume, students were encouraged to use libraries of subroutines to complete programming assignments.

To summarise the importance, not only of these books, but of Todd’s contribution in general, we give a quotation from A. S. Householder’s review of [40]:

“Probably no one has a practical and theoretical background surpassing that of the author, and this book is altogether unique”.

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FIGURE 2. John Todd and Olga Taussky-Todd, March 1973, Los Angeles. (Source: Frank Uhlig)

ystème d'équations linéaires en nombre inférieur à celui des inconnues.- Application de la méthode à la résolution d'un système défini d'équations linéaires (Procédé du Commandant Cholesky). *Bull. Géodésique*, 2:67–77, 1924.

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