Wrinkles and Creases in Bending and Unbending of a Cylindrical Sector

Taisiya Sigaeva[1], Les Sudak[1]
Michel Destrade[2], **Robert Mangan**[2], Luigi Vergori[3]

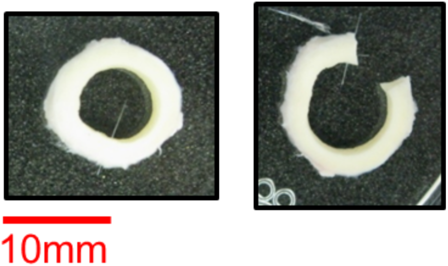
[1]Department of Mechanical and Manufacturing Engineering, University of Calgary, Canada

[2]School of Mathematics, Statistics and Applied Mathematics, NUI Galway, Ireland

[3]Dipartimento di Ingegneria, Università degli studi di Perugia, Italy

Biomechanical Motivation

- Opening Angle Method (Taber):



- Aortic Valve Leaflet:

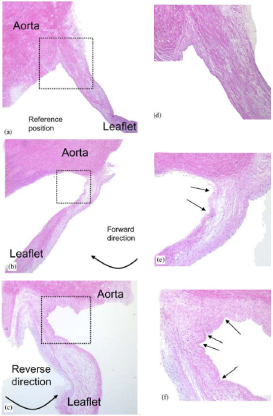


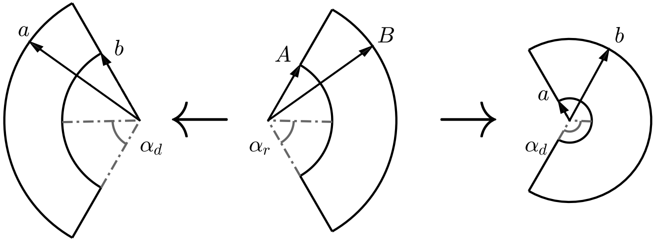
Figure: Mirnajafi A. et al. 2006. The flexural rigidity of the aortic valve leaflet in the commissural region. *J. Biomech.*

Preliminaries

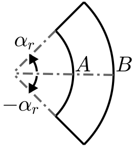
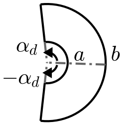
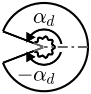
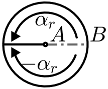
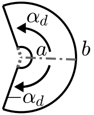
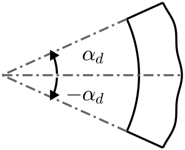
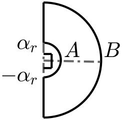
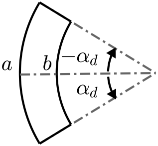
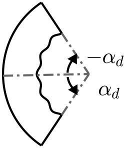
Deformation

$$\begin{cases} r = r(R), \\ \theta = \kappa\Theta, \\ z = \lambda_z Z. \end{cases}$$

$\lambda_z = \ell/L$	axial stretch;
$\kappa = \alpha_d/\alpha_r$	change in the angles;
$\alpha_r \in (0, \pi]$	undeformed angle;
$\alpha_d \in [-\pi, \pi] \setminus \{0\}$	deformation angle;
A, B	undeformed radii;
$r(A) = a, r(B) = b$	deformed radii.



Possible Scenarios

	Undeformed	Deformed	Buckled
Bending $\alpha_d > \alpha_r > 0$ $\kappa > 1$			
Unbending $0 < \alpha_d < \alpha_r$ $0 < \kappa < 1$			
Eversion $\alpha_d < 0 < \alpha_r$ $\kappa < 0$			

Stroh formulation

$$\mathbf{u} = [U(r)\mathbf{e}_r + V(r)\mathbf{e}_\theta]\mathrm{e}^{\mathrm{i}n\theta}, \qquad n = \frac{m\pi}{\alpha_d} = \frac{m\pi}{\kappa\alpha_r}\ (m \in \mathbb{N}),$$

$$\dot{\mathbf{S}}^{\mathrm{T}}\mathbf{e}_r = [S_{rr}(r)\mathbf{e}_r + S_{r\theta}(r)\mathbf{e}_\theta]\mathrm{e}^{\mathrm{i}n\theta}, \quad S_{rr}(a) = S_{rr}(b) = S_{r\theta}(a) = S_{r\theta}(b) = 0.$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\boldsymbol{\eta}(r) = \frac{\mathrm{i}}{r}\mathbf{G}(r)\boldsymbol{\eta}(r), \quad \boldsymbol{\eta} = [U, V, \mathrm{i}rSrr, \mathrm{i}rS_{r\theta}]^{\mathrm{T}}$$

$$\mathbf{G} = \begin{pmatrix} \mathrm{i} & -n & 0 & 0 \\ -n(1-\sigma_1/\alpha) & -\mathrm{i}(1-\sigma_1/\alpha) & 0 & -1/\alpha \\ \kappa_{11} & \mathrm{i}\kappa_{12} & -\mathrm{i} & -n(1-\sigma_1/\alpha) \\ -\mathrm{i}\kappa_{12} & \kappa_{22} & -n & \mathrm{i}(1-\sigma_1/\alpha) \end{pmatrix}$$

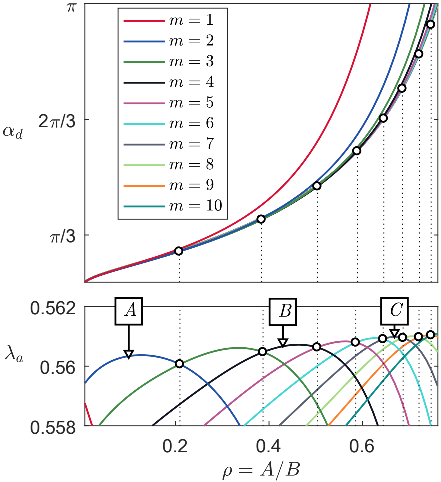
$$\alpha = \frac{\lambda\widehat{W}'(\lambda)}{\lambda^4 - 1}, \quad \gamma = \lambda^4\alpha, \quad \beta = \frac{\lambda^2}{2}\widehat{W}''(\lambda) - \alpha.$$

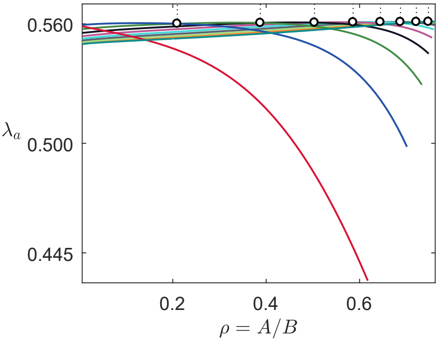
$$\kappa_{11} = 2(\alpha + \beta - \sigma_1) + n^2[\gamma - \alpha(1-\sigma_1/\alpha)^2],$$

$$\kappa_{12} = n[2\beta + \alpha + \gamma - {\sigma_1}^2/\alpha],$$

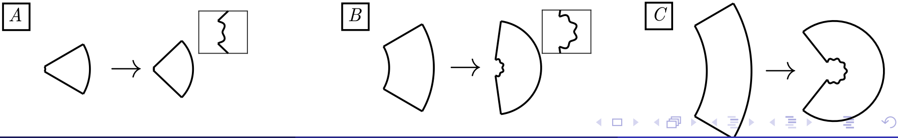
$$\kappa_{22} = \gamma - \alpha(1-\sigma_1/\alpha)^2 + 2n^2(\alpha + \beta - \sigma_1).$$

Numerical results: $\alpha_{\mathsf{r}} = \pi/6$, bending

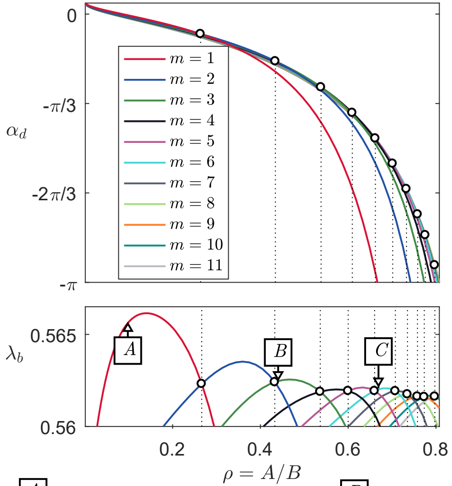


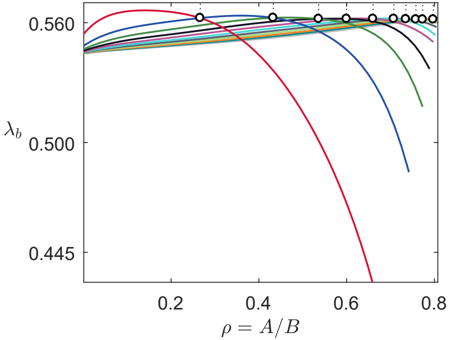


Sector	$\rho = A/B$	m	α_d	λ_a
$\boxed{A}$	0.1	2	0.24π	0.560
$\boxed{B}$	0.486	4	0.458π	0.560
$\boxed{C}$	0.67	7	0.718π	0.561

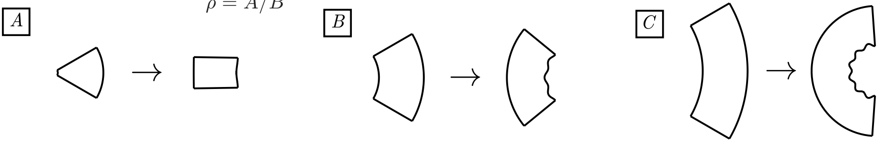


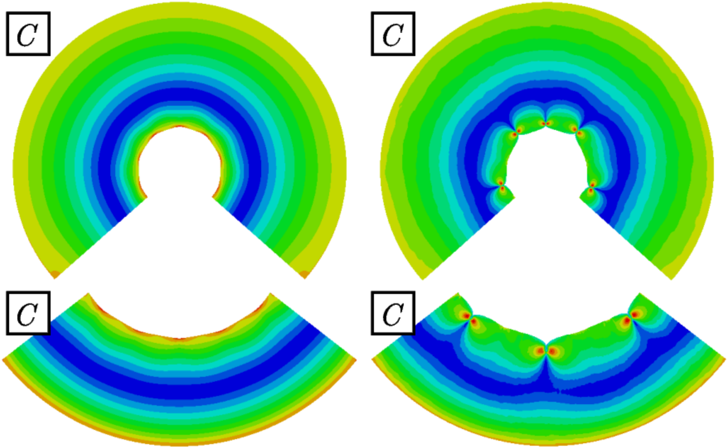
Numerical results: $\alpha_r = \pi/6$, unbending-eversion



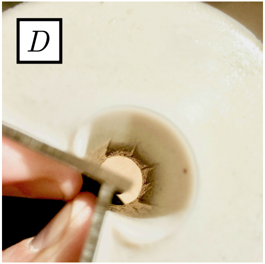


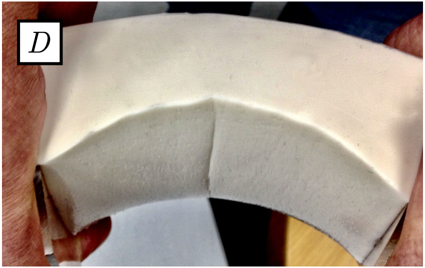
Sector	$\rho = A/B$	m	α_d	λ_b
A	0.1	1	-0.005π	0.565
B	0.486	3	-0.22π	0.561
C	0.67	6	-0.48π	0.562




C
C
C
C

Experiments





Future work

- Different material model;
- Multilayered;
- Anisotropy.

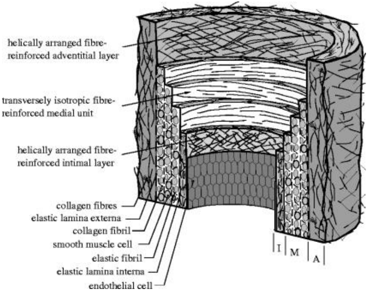


Figure: Gasser, T.C., Ogden, R.W. and Holzapfel, G.A., 2006. Hyperelastic modelling of arterial layers with distributed collagen fibre orientations. Journal of the royal society interface, 3(6), pp.15-35.